

- 3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtered complete lattice filtrator with down-aligned, binarily meet-closed, separable core which is a complete boolean lattice.
- 4°. $a^* = \overline{\text{Cor } a} = \overline{\text{Cor}' a}$ for every $a \in \mathfrak{A}$.

PROOF.

1° \Rightarrow 2°. Obvious.

2° \Rightarrow 3°. It is filtered by theorem 531. It is complete lattice filtrator by 515. It is with binarily meet-closed core (proposition 533), with separable core (theorem 534).

3° \Rightarrow 4°. Our filtrator is with join-closed core (theorem 531). $a^* = \bigsqcup^{\mathfrak{A}} \left\{ \frac{c \in \mathfrak{A}}{c \sqcap^{\mathfrak{A}} a = \perp^{\mathfrak{A}}} \right\}$. But $c \sqcap^{\mathfrak{A}} a = \perp^{\mathfrak{A}} \Rightarrow \exists C \in \text{up } c : C \sqcap^{\mathfrak{A}} a = \perp^{\mathfrak{A}}$. So

$$\begin{aligned}
 a^* &= \\
 &= \bigsqcup^{\mathfrak{A}} \left\{ \frac{C \in \mathfrak{Z}}{C \sqcap^{\mathfrak{A}} a = \perp^{\mathfrak{A}}} \right\} = \\
 &= \bigsqcup^{\mathfrak{A}} \left\{ \frac{C \in \mathfrak{Z}}{a \sqsubseteq \overline{C}} \right\} = \\
 &= \bigsqcup^{\mathfrak{A}} \left\{ \frac{\overline{C}}{C \in \mathfrak{Z}, a \sqsubseteq C} \right\} = \\
 &= \bigsqcup^{\mathfrak{A}} \left\{ \frac{\overline{C}}{C \in \text{up } a} \right\} = \\
 &= \bigsqcup^{\mathfrak{Z}} \left\{ \frac{\overline{C}}{C \in \text{up } a} \right\} = \\
 &= \overline{\bigsqcup^{\mathfrak{Z}} \left\{ \frac{C}{C \in \text{up } a} \right\}} = \\
 &= \overline{\bigsqcup^{\mathfrak{Z}} \text{up } a} = \\
 &= \overline{\text{Cor } a}
 \end{aligned}$$

(used lemma 548).

$\text{Cor } a = \overline{\text{Cor}' a}$ by theorem 542. □

THEOREM 592. The following is an implications tuple:

- 1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
- 2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a complete boolean lattice.
- 3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtered down-aligned and up-aligned complete lattice filtrator with binarily meet-closed, separable and co-separable core which is a complete boolean lattice.
- 4°. $a^* = a^+ = \overline{\text{Cor } a} = \overline{\text{Cor}' a} \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.

PROOF.

1° \Rightarrow 2°. Obvious.

2° \Rightarrow 3°. The filtrator $(\mathfrak{A}, \mathfrak{Z})$ is filtered by the theorem 531. \mathfrak{A} is a complete lattice by corollary 515. $(\mathfrak{A}, \mathfrak{Z})$ is with co-separable core by theorem 587. $(\mathfrak{A}, \mathfrak{Z})$ is binarily meet-closed by proposition 533, with separable core by theorem 534.

3° \Rightarrow 4°. Comparing two last theorems. □

THEOREM 593. The following is an implications tuple:

- 1°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a complete lattice.