PROOF. For every $A, B \in \mathfrak{Z}$

$$\begin{aligned} A \sqcup^{\mathfrak{Z}} B \in \partial a \Leftrightarrow \\ A \sqcup^{\mathfrak{A}} B \in \partial a \Leftrightarrow \\ (A \sqcup^{\mathfrak{A}} B) \sqcap^{\mathfrak{A}} a \neq \bot^{\mathfrak{A}} \Leftrightarrow \\ (A \sqcap^{\mathfrak{A}} a) \sqcup^{\mathfrak{A}} (B \sqcap^{\mathfrak{A}} a) \neq \bot^{\mathfrak{A}} \Leftrightarrow \\ A \sqcap^{\mathfrak{A}} a \neq \bot^{\mathfrak{A}} \vee B \sqcap^{\mathfrak{A}} a \neq \bot^{\mathfrak{A}} \Leftrightarrow \\ A \in \partial a \vee B \in \partial a. \end{aligned}$$

That ∂a doesn't contain $\bot^{\mathfrak{A}}$ is obvious. □

DEFINITION 552. I call a filtrator *star-separable* when its core is a separation subset of its base.

5.14. Atomic Elements of a Filtrator

See [4, 9] for more detailed treatment of ultrafilters and prime filters.

PROPOSITION 553. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a meet-semilattice with greatest element.
3°. $\mathfrak{A}$ is a complete lattice.
4°. $\operatorname{atoms} \bigsqcap S = \bigcap \langle \operatorname{atoms} \rangle^* S$ for every $S \in \mathscr{P}\mathfrak{A}$.
5°. $\operatorname{atoms}(a \sqcap b) = \operatorname{atoms} a \cap \operatorname{atoms} b$ for $a, b \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. Corollary 515.
3°⇒4°. Theorem 108.
4°⇒5°. Obvious.

□

PROPOSITION 554. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a distributive lattice which is and ideal base.
3°. $\mathfrak{A}$ is a starrish join-semilattice.
4°. $\operatorname{atoms}(a \sqcup b) = \operatorname{atoms} a \cup \operatorname{atoms} b$ for $a, b \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. Corollary 528.
3°⇒4°. Corollary 493.

□

THEOREM 555. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a meet-semilattice.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtered weakly down-aligned filtrator with binarily meet-closed core $\mathfrak{Z}$ which is a meet-semilattice.
4°. a is an atom of $\mathfrak{Z}$ iff $a \in \mathfrak{Z}$ and a is an atom of $\mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. It is filtered by the theorem 531, binarily meet-closed by corollary 533.