COROLLARY 526. If $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator and $\mathfrak{Z}$ is a lattice, then $\mathfrak{A}$ is a lattice.

5.8.2. Distributivity of the Lattice of Filters.

THEOREM 527. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a distributive lattice.
3°. $\mathcal{A} \sqcup^{\mathfrak{A}} \prod^{\mathfrak{A}} S = \prod^{\mathfrak{A}} \langle \mathcal{A} \sqcup^{\mathfrak{A}} \rangle^* S$ for $S \in \mathscr{P}\mathfrak{A}$ and $\mathcal{A} \in \mathfrak{A}$.

PROOF.
1°⇒2°. Obvious.
2°⇒3°. Taking into account the previous section, we have:

$$\operatorname{up}\left(\mathcal{A} \sqcup^{\mathfrak{A}} \prod^{\mathfrak{A}} S\right) =$$

$$\operatorname{up} \mathcal{A} \cap \operatorname{up} \prod^{\mathfrak{A}} S =$$

$$\operatorname{up} \mathcal{A} \cap \left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \bigcup \langle \operatorname{up} \rangle^* S \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n \in \operatorname{up} \mathcal{A}, K_i \in \bigcup \langle \operatorname{up} \rangle^* S \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \operatorname{up} \mathcal{A}, K_i \in \bigcup \langle \operatorname{up} \rangle^* S \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \operatorname{up} \mathcal{A} \cap \bigcup \langle \operatorname{up} \rangle^* S \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \bigcup \langle \operatorname{up} \mathcal{A} \cap \rangle^* \langle \operatorname{up} \rangle^* S \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \bigcup \left\{ \frac{\operatorname{up} \mathcal{A} \cap \operatorname{up} \mathcal{X}}{\mathcal{X} \in S} \right\} \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\left\{ \frac{K_0 \sqcap^{\mathfrak{Z}} \dots \sqcap^{\mathfrak{Z}} K_n}{K_i \in \bigcup \left\{ \frac{\operatorname{up}(\mathcal{A} \sqcup^{\mathfrak{A}} \mathcal{X})}{\mathcal{X} \in S} \right\} \text{ where } i = 0, \dots, n \text{ for } n \in \mathbb{N}} \right\} =$$

$$\operatorname{up} \prod^{\mathfrak{A}} \left\{ \frac{\mathcal{A} \sqcup^{\mathfrak{A}} \mathcal{X}}{\mathcal{X} \in S} \right\} =$$

$$\operatorname{up} \prod^{\mathfrak{A}} \langle \mathcal{A} \sqcup^{\mathfrak{A}} \rangle^* S.$$

□

COROLLARY 528. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a distributive lattice which is an ideal base.
3°. $\mathfrak{A}$ is a distributive and co-brouwerian lattice.

COROLLARY 529. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a distributive lattice with greatest element.