- Filters are nonempty sets F with $A \sqcap B \in F \Leftrightarrow A \in F \wedge B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a meet-semilattice.
- Ideals are nonempty sets F with $A \sqcup B \in F \Leftrightarrow A \in F \wedge B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a join-semilattice.
- Free stars are sets F not equal to $\mathscr{P}\mathfrak{Z}$ with $A \sqcup B \in F \Leftrightarrow A \in F \vee B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a join-semilattice.
- Mixers are sets F not equal to $\mathscr{P}\mathfrak{Z}$ with $A \sqcap B \in F \Leftrightarrow A \in F \vee B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a meet-semilattice.

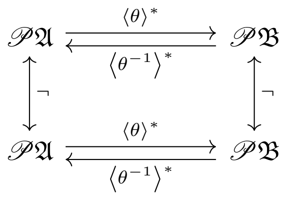
By duality and and an above theorem about filters, we have:

Proposition 476.

- Filters are nonempty upper sets F with $A \sqcap B \in F \Leftarrow A \in F \wedge B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a meet-semilattice.
- Ideals are nonempty lower sets F with $A \sqcup B \in F \Leftarrow A \in F \wedge B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a join-semilattice.
- Free stars are upper sets F not equal to $\mathscr{P}\mathfrak{Z}$ with $A \sqcup B \in F \Rightarrow A \in F \vee B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a join-semilattice.
- Mixers are lower sets F not equal to $\mathscr{P}\mathfrak{Z}$ with $A \sqcap B \in F \Rightarrow A \in F \vee B \in F$ (for every $A, B \in \mathfrak{Z}$), whenever $\mathfrak{Z}$ is a meet-semilattice.

5.6.5. The general diagram. Let $\mathfrak{A}$ and $\mathfrak{B}$ be two posets connected by an order reversing isomorphism $\theta : \mathfrak{A} \to \mathfrak{B}$. We have commutative diagram on the figure 1 in the category **Set**:

Figure 1.



Theorem 477. This diagram is commutative, every arrow of this diagram is an isomorphism, every cycle in this diagrams is an identity (therefore “parallel” arrows are mutually inverse).

Proof. That every arrow is an isomorphism is obvious.
Show that $\langle\theta\rangle^* \neg X = \neg\langle\theta\rangle^* X$ for every set $X \in \mathscr{P}\mathfrak{A}$.
Really,

$$p \in \langle\theta\rangle^* \neg X \Leftrightarrow \exists q \in \neg X : p = \theta q \Leftrightarrow \exists q \in \neg X : \theta^{-1} p = q \Leftrightarrow \theta^{-1} p \in \neg X \Leftrightarrow \nexists q \in X : q = \theta^{-1} p \Leftrightarrow \nexists q \in X : \theta q = p \Leftrightarrow p \notin \langle\theta\rangle^* X \Leftrightarrow p \in \neg\langle\theta\rangle^* X.$$

Thus the theorem follows from lemma 194. □

This diagram can be restricted to filters, ideals, free stars, and mixers, see figure 2:

Theorem 478. It is a restriction of the above diagram. Every arrow of this diagram is an isomorphism, every cycle in these diagrams is an identity. (To prove that, is an easy application of duality and the above lemma.)