

OBVIOUS 393. Restriction of a **Rel**-morphism is **Rel**-morphism.

OBVIOUS 394. $f|_A = f \sqcap (A \times \top^{\mathcal{T} \text{Dst } f})$ for every **Rel**-morphism f and $A \in \mathcal{T} \text{Src } f$.

OBVIOUS 395. $\langle f \rangle^* X = \langle f \rangle^*(X \sqcap \text{dom } f) = \text{im}(f|_X)$ for every **Rel**-morphism f and $X \in \mathcal{T} \text{Src } f$.

OBVIOUS 396. $f \sqsubseteq A \times B \Leftrightarrow \text{dom } f \sqsubseteq A \wedge \text{im } f \sqsubseteq B$ for every **Rel**-morphism f and $A \in \mathcal{T} \text{Src } f, B \in \mathcal{T} \text{Dst } f$.

THEOREM 397. Let A, B be sets. If $S \in \mathcal{P}(\mathcal{T}A \times \mathcal{T}B)$ then

$$\bigsqcap_{(A,B) \in S} (A \times B) = \bigsqcap \text{dom } S \times \bigsqcap \text{im } S.$$

PROOF. For every atomic $x \in \mathcal{T}A, y \in \mathcal{T}B$ we have

$$\begin{aligned} x \times y \sqsubseteq \bigsqcap_{(A,B) \in S} (A \times B) &\Leftrightarrow \forall (A,B) \in S : x \times y \sqsubseteq A \times B \Leftrightarrow \\ \forall (A,B) \in S : (x \sqsubseteq A \wedge y \sqsubseteq B) &\Leftrightarrow \forall A \in \text{dom } S : x \sqsubseteq A \wedge \forall B \in \text{im } S : y \sqsubseteq B \Leftrightarrow \\ x \sqsubseteq \bigsqcap \text{dom } S \wedge y \sqsubseteq \bigsqcap \text{im } S &\Leftrightarrow x \times y \sqsubseteq \bigsqcap \text{dom } S \times \bigsqcap \text{im } S. \end{aligned}$$

□

OBVIOUS 398. If U, W are sets and $A \in \mathcal{T}(U)$ then $A \times$ is a complete homomorphism from the lattice $\mathcal{T}(W)$ to the lattice **Rel**(U, W), if also $A \neq \perp$ then it is an order embedding.