PROOF.

$$X\ [g\circ f]^*\ Z \Leftrightarrow \exists x\in X, z\in Z : x\ (g\circ f)\ z \Leftrightarrow \\ \exists x\in X, z\in Z, \beta : (x\ f\ \beta \wedge \beta\ g\ z) \Leftrightarrow \\ \exists \beta : (\exists x\in X : x\ f\ \beta \wedge \exists y\in Y : \beta\ g\ z) \Leftrightarrow \exists\beta : (X\ [f]^*\ \{\beta\} \wedge \{\beta\}\ [g]^*\ Z).$$

□

COROLLARY 382. $X\ [g\circ f]^*\ Z \Leftrightarrow \exists y\in \operatorname{atoms}^{\mathscr{T} B} : (X\ [f]^*\ y \wedge y\ [g]^*\ Z)$ for $f\in \mathbf{Rel}(A,B)$, $g\in\mathbf{Rel}(B,C)$ (for sets A, B, C).

PROPOSITION 383. $f\circ \bigcup G = \bigcup_{g\in G}(f\circ g)$ and $\bigcup G\circ f = \bigcup_{g\in G}(g\circ f)$ for every binary relation f and set G of binary relations.

PROOF. We will prove only $\bigcup G\circ f = \bigcup_{g\in G}(g\circ f)$ as the other formula follows from duality. Really

$$(x,z)\in \bigcup G\circ f \Leftrightarrow \exists y : ((x,y)\in f \wedge (y,z)\in \bigcup G) \Leftrightarrow \\ \exists y, g\in G : ((x,y)\in f\wedge (y,z)\in g) \Leftrightarrow \exists g\in G : (x,z)\in g\circ f \Leftrightarrow (x,z)\in \bigcup_{g\in G}(g\circ f).$$

□

COROLLARY 384. Every **Rel**-morphism is metacomplete and co-metacomplete.

PROPOSITION 385. The following are equivalent for a **Rel**-morphism f:

1°. f is monovalued.
2°. f is metamonovalued.
3°. f is weakly metamonovalued.
4°. $\langle f\rangle^* a$ is either atomic or least whenever $a\in \operatorname{atoms}^{\mathscr{T}\operatorname{Src} f}$.
5°. $\langle f^{-1}\rangle^*(I\sqcap J) = \langle f^{-1}\rangle^* I \sqcap \langle f^{-1}\rangle^* J$ for every $I,J\in\mathscr{T}\operatorname{Src} f$.
6°. $\langle f^{-1}\rangle^* \bigsqcap S = \bigsqcap_{Y\in S}\langle f^{-1}\rangle^* Y$ for every $S\in \mathscr{P}\mathscr{T}\operatorname{Src} f$.

PROOF.

2°⇒3°. Obvious.
1°⇒2°. Take $x\in \operatorname{atoms}^{\mathscr{T}\operatorname{Src} f}$; then $fx\in \operatorname{atoms}^{\mathscr{T}\operatorname{Dst} f}\cup\{\bot^{\mathscr{T}\operatorname{Dst} f}\}$ and thus

$$\left\langle \left(\bigsqcap G\right)\circ f\right\rangle^* x = \left\langle \bigsqcap G\right\rangle^* \langle f\rangle^* x = \bigsqcap_{g\in G}\langle g\rangle^*\langle f\rangle^* x = \\ \bigsqcap_{g\in G}\langle g\circ f\rangle^* x = \left\langle \bigsqcap_{g\in G}(g\circ f)\right\rangle^* x;$$

so $(\bigsqcap G)\circ f = \bigsqcap_{g\in G}(g\circ f)$.
3°⇒1°. Take $g=\{(a,y)\}$ and $h=\{(b,y)\}$ for arbitrary $a\neq b$ and arbitrary y. We have $g\cap h=\emptyset$; thus $(g\circ f)\cap(h\circ f) = (g\cap h)\circ f=\bot$ and thus impossible $x\ f\ a\wedge x\ f\ b$ as otherwise $(x,y)\in(g\circ f)\cap(h\circ f)$. Thus f is monovalued.