CHAPTER 4

Typed sets and category Rel

4.1. Relational structures

DEFINITION 352. A *relational structure* is a pair consisting of a set and a tuple of relations on this set.

A poset $(\mathfrak{A}, \sqsubseteq)$ can be considered as a relational structure: $(\mathfrak{A}, \llbracket \sqsubseteq \rrbracket)$.

A set can X be considered as a relational structure with zero relations: $(X, \llbracket\rrbracket)$.

This book is not about relational structures. So I will not introduce more examples.

Think about relational structures as a common place for sets or posets, as far as they are considered in this book.

We will denote $x \in (\mathfrak{A}, R)$ iff $x \in \mathfrak{A}$ for a relational structure $(\mathfrak{A}, R)$.

4.2. Typed elements and typed sets

We sometimes want to differentiate between the same element of two different sets. For example, we may want to consider different the natural number 3 and the rational number 3. In order to describe this in a formal way we consider elements of sets together with sets themselves. For example, we can consider the pairs $(\mathbb{N}, 3)$ and $(\mathbb{Q}, 3)$.

DEFINITION 353. A *typed element* is a pair $(\mathfrak{A}, a)$ where $\mathfrak{A}$ is a relational structure and $a \in \mathfrak{A}$.

I denote $\operatorname{type}(\mathfrak{A}, a) = \mathfrak{A}$ and $\operatorname{GR}(\mathfrak{A}, a) = a$.

DEFINITION 354. I will denote typed element $(\mathfrak{A}, a)$ as $@^{\mathfrak{A}} a$ or just $@a$ when $\mathfrak{A}$ is clear from context.

DEFINITION 355. A *typed set* is a typed element equal to $(\mathscr{P}U, A)$ where U is a set and A is its subset.

REMARK 356. *Typed sets* is an awkward formalization of type theory sets in ZFC (U is meant to express the *type* of the set). This book could be better written using type theory instead of ZFC, but I want my book to be understandable for everyone knowing ZFC. $(\mathscr{P}U, A)$ should be understood as a set A of type U. For an example, consider $(\mathscr{P}\mathbb{R}, [0; 10])$; it is the closed interval $[0; 10]$ whose elements are considered as real numbers.

DEFINITION 357. $\mathfrak{T}\mathfrak{A} = \left\{ \frac{(\mathfrak{A}, a)}{a \in \mathfrak{A}} \right\} = \{\mathfrak{A}\} \times \mathfrak{A}$ for every relational structure $\mathfrak{A}$.

REMARK 358. $\mathfrak{T}\mathfrak{A}$ is the set of typed elements of $\mathfrak{A}$.

DEFINITION 359. If $\mathfrak{A}$ is a poset, we introduce order on its typed elements isomorphic to the order of the original poset: $(\mathfrak{A}, a) \sqsubseteq (\mathfrak{A}, b) \Leftrightarrow a \sqsubseteq b$.

DEFINITION 360. I denote $\operatorname{GR}(\mathfrak{A}, a) = a$ for a typed element $(\mathfrak{A}, a)$.

DEFINITION 361. I will denote *typed subsets* of a typed poset $(\mathscr{P}U, A)$ as $\mathscr{P}(\mathscr{P}U, A) = \left\{ \frac{(\mathscr{P}U, X)}{X \in \mathscr{P}A} \right\} = \{\mathscr{P}U\} \times \mathscr{P}A$.