PROOF. Let $L' \in \left\{ \frac{L \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i}}{\operatorname{curry}(L) \in \prod(\mathrm{GR} \circ F)} \right\}$. Then $L' \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i}$ and $\operatorname{curry}(L') \in \prod(\mathrm{GR} \circ F)$.

Let $P = \lambda i \in \operatorname{dom} F : L'|_{\{i\} \times \operatorname{arity} F_i}$. Then $P \in \coprod_{i\in\operatorname{dom} F} \mho^{\{i\}\times\operatorname{arity} F_i}$ and $\bigcup \operatorname{im} P = L'$. So $L' \in \left\{ \frac{\bigcup \operatorname{im} P}{P \in \coprod_{i\in\operatorname{dom} F} \mho^{\{i\}\times\operatorname{arity} F_i} \wedge \operatorname{curry}(\bigcup \operatorname{im} P) \in \prod(\mathrm{GR} \circ F)} \right\}$.

Let now $L' \in \left\{ \frac{\bigcup \operatorname{im} P}{P \in \coprod_{i\in\operatorname{dom} F} \mho^{\{i\}\times\operatorname{arity} F_i} \wedge \operatorname{curry}(\bigcup \operatorname{im} P) \in \prod(\mathrm{GR} \circ F)} \right\}$. Then there exists $P \in \coprod_{i\in\operatorname{dom} F} \mho^{\{i\}\times\operatorname{arity} F_i}$ such that $L' = \bigcup \operatorname{im} P$ and $\operatorname{curry}(L') \in \prod(\mathrm{GR} \circ F)$. Evidently $L' \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i}$. So $L' \in \left\{ \frac{L \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i}}{\operatorname{curry}(L) \in \prod(\mathrm{GR} \circ F)} \right\}$. □

LEMMA 307. $\left\{ \frac{f \circ \bigoplus(\operatorname{arity} \circ F)}{f \in \mathrm{GR} \prod^{(\mathrm{ord})} F} \right\} = \left\{ \frac{\bigcup \operatorname{im} P}{P \in \prod_{i\in\operatorname{dom} F} F'_i} \right\}$.

PROOF.

$$L \in \left\{ \frac{\bigcup \operatorname{im} P}{P \in \prod_{i\in\operatorname{dom} F} F'_i} \right\} \Leftrightarrow$$

$$L \in \left\{ \frac{\bigcup \operatorname{im} P}{P \in \coprod_{i\in\operatorname{dom} F} \mho^{\{i\}\times\operatorname{arity} F_i} \wedge \operatorname{curry}(\bigcup \operatorname{im} P) \in \prod(\mathrm{GR} \circ F)} \right\} \Leftrightarrow$$

$$L \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i} \wedge \operatorname{curry}(L) \in \prod(\mathrm{GR} \circ F) \Leftrightarrow$$

$$L \in \mho^{\coprod_{i\in\operatorname{dom} F} \operatorname{arity} F_i} \wedge \operatorname{curry}(L) \in \left\{ \frac{\operatorname{curry}(f) \circ \bigoplus(\operatorname{arity} \circ F)}{f \in \mathrm{GR} \prod^{(\mathrm{ord})} F} \right\} \Leftrightarrow$$

(because $\bigoplus(\operatorname{arity} \circ F)$ is a bijection)

$$\operatorname{curry}(L) \circ \left(\bigoplus(\operatorname{arity} \circ F) \right)^{-1} \in \left\{ \frac{\operatorname{curry}(f)}{f \in \mathrm{GR} \prod^{(\mathrm{ord})} F} \right\} \Leftrightarrow$$

$$L \circ \left(\bigoplus(\operatorname{arity} \circ F) \right)^{-1} \in \left\{ \frac{f}{f \in \mathrm{GR} \prod^{(\mathrm{ord})} F} \right\} \Leftrightarrow$$

(because $\bigoplus(\operatorname{arity} \circ F)$ is a bijection)

$$L \in \left\{ \frac{f \circ \bigoplus(\operatorname{arity} \circ F)}{f \in \mathrm{GR} \prod^{(\mathrm{ord})} F} \right\}.$$

□

THEOREM 308. $\mathrm{GR} \prod^{(\mathrm{ord})} F = \left\{ \frac{(\bigcup \operatorname{im} P) \circ (\bigoplus(\operatorname{arity} \circ F))^{-1}}{P \in \prod_{i\in\operatorname{dom} F} F'_i} \right\}$.

PROOF. From the lemma, because $\bigoplus(\operatorname{arity} \circ F)$ is a bijection. □

THEOREM 309. $\mathrm{GR} \prod^{(\mathrm{ord})} F = \left\{ \frac{\bigcup_{i\in\operatorname{dom} F} \left(P_i \circ (\bigoplus(\operatorname{arity} \circ f))^{-1} \right)}{P \in \prod_{i\in\operatorname{dom} F} F'_i} \right\}$.

PROOF. From the previous theorem. □

THEOREM 310. $\mathrm{GR} \prod^{(\mathrm{ord})} F = \left\{ \frac{\bigcup \operatorname{im} P}{P \in \prod_{i\in\operatorname{dom} F} \left\{ \frac{f \circ \left(\bigoplus(\operatorname{arity} \circ f) \right)^{-1}}{f \in F'_i} \right\}} \right\}$.

PROOF. From the previous. □