family indexed by an ordinal n. Then $f(x)$ can be taken. The same function f can take different number of arguments. (See below for the exact definition.)

Some of such functions f are associative in the sense defined below. If a function is associative in the below defined sense, then the binary operation induced by this function is associative in the usual meaning of the word "associativity" as defined in basic algebra.

I also introduce and research an important example of infinitely associative function, which I call *ordinated product*.

Note that my searching about infinite associativity and ordinals in Internet has provided no useful results. As such there is a reason to assume that my research of generalized associativity in terms of ordinals is novel.

3.7.2. Used notation. We identify natural numbers with finite Von Neumann's ordinals (further just *ordinals* or *ordinal numbers*).

For simplicity we will deal with small sets (members of a Grothendieck universe). We will denote the Grothendieck universe (aka *universal set*) as $\mho$.

I will denote a tuple of n elements like $[\![a_0, \dots, a_{n-1}]\!]$. By definition

$$[\![a_0, \dots, a_{n-1}]\!] = \{(0, a_0), \dots, (n-1, a_{n-1})\}.$$

Note that an ordered pair (a, b) is not the same as the tuple $[\![a, b]\!]$ of two elements. (However, we will use them interchangeably.)

DEFINITION 281. An *anchored relation* is a tuple $[\![n, r]\!]$ where n is an index set and r is an n-ary relation.

For an anchored relation $\operatorname{arity}[\![n, r]\!] = n$. The graph[1] of $[\![n, r]\!]$ is defined as follows: $\operatorname{GR}[\![n, r]\!] = r$.

DEFINITION 282. $\Pr_i f$ is a function defined by the formula

$$\Pr_i f = \left\{ \frac{x_i}{x \in f} \right\}$$

for every small n-ary relation f where n is an ordinal number and $i \in n$. Particularly for every n-ary relation f and $i \in n$ where $n \in \mathbb{N}$

$$\Pr_i f = \left\{ \frac{x_i}{[\![x_0, \dots, x_{n-1}]\!] \in f} \right\}.$$

Recall that Cartesian product is defined as follows:

$$\prod a = \left\{ \frac{z \in (\bigcup \operatorname{im} a)^{\operatorname{dom} a}}{\forall i \in \operatorname{dom} a : z(i) \in a_i} \right\}.$$

OBVIOUS 283. If a is a small function, then $\prod a = \left\{ \frac{z \in \mho^{\operatorname{dom} a}}{\forall i \in \operatorname{dom} a : z(i) \in a_i} \right\}$.

3.7.2.1. *Currying and uncurrying.*

The customary definition. Let X, Y, Z be sets.

We will consider variables $x \in X$ and $y \in Y$.

Let a function $f \in Z^{X \times Y}$. Then $\operatorname{curry}(f) \in (Z^Y)^X$ is the function defined by the formula $(\operatorname{curry}(f)x)y = f(x, y)$.

Let now $f \in (Z^Y)^X$. Then $\operatorname{uncurry}(f) \in Z^{X \times Y}$ is the function defined by the formula $\operatorname{uncurry}(f)(x, y) = (fx)y$.

OBVIOUS 284.

1°. $\operatorname{uncurry}(\operatorname{curry}(f)) = f$ for every $f \in Z^{X \times Y}$.
2°. $\operatorname{curry}(\operatorname{uncurry}(f)) = f$ for every $f \in (Z^Y)^X$.

[1]It is unrelated with graph theory.