DEFINITION 270. A morphism f of a partially ordered category is *weakly co-metacomplete* when $(g \sqcup h) \circ f = (g \circ f) \sqcup (h \circ f)$ whenever g and h are morphisms with a suitable domain and image.

OBVIOUS 271.

1°. Metamonovalued morphisms are weakly metamonovalued.
2°. Metainjective morphisms are weakly metainjective.
3°. Metacomplete morphisms are weakly metacomplete.
4°. Co-metacomplete morphisms are weakly co-metacomplete.

3.5. Partitioning

DEFINITION 272. Let $\mathfrak{A}$ be a complete lattice. *Torning* of an element $a \in \mathfrak{A}$ is a set $S \in \mathscr{P}\mathfrak{A} \setminus \{\bot\}$ such that

$$\bigsqcup S = a \quad \text{and} \quad \forall x, y \in S : (x \neq y \Rightarrow x \asymp y).$$

DEFINITION 273. Let $\mathfrak{A}$ be a complete lattice. *Weak partition* of an element $a \in \mathfrak{A}$ is a set $S \in \mathscr{P}\mathfrak{A} \setminus \{\bot\}$ such that

$$\bigsqcup S = a \quad \text{and} \quad \forall x \in S : x \asymp \bigsqcup (S \setminus \{x\}).$$

DEFINITION 274. Let $\mathfrak{A}$ be a complete lattice. *Strong partition* of an element $a \in \mathfrak{A}$ is a set $S \in \mathscr{P}\mathfrak{A} \setminus \{\bot\}$ such that

$$\bigsqcup S = a \quad \text{and} \quad \forall A, B \in \mathscr{P}S : (A \asymp B \Rightarrow \bigsqcup A \asymp \bigsqcup B).$$

OBVIOUS 275.

1°. Every strong partition is a weak partition.
2°. Every weak partition is a torning.

DEFINITION 276. *Complete lattice generated by* a set P (on a complete lattice) is the set (obviously having the structure of complete lattice) $P_0 \cup P_1 \cup \ldots$ where $P_0 = P$ and $P_{i+1} = \left\{ \frac{\bigsqcup K, \bigsqcap K}{K \in \mathscr{P} P_i} \right\}$.

OBVIOUS 277. Complete lattice generated by a set is indeed a complete lattice.

EXAMPLE 278. $[S] \neq \left\{ \frac{\bigsqcup^{\mathfrak{A}} X}{X \in \mathscr{P} S} \right\}$, where $[S]$ is the complete lattice generated by a strong partition S of a filter on a set.

PROOF. Consider any infinite set U and its strong partition $S = \left\{ \frac{\uparrow^U \{x\}}{x \in U} \right\}$. The set S consists only of principal filters. But $[S]$ contains (exercise!) some nonprincipal filters. □

By the way:

PROPOSITION 279. $\left\{ \frac{\bigsqcup^{\mathfrak{A}} X}{X \in \mathscr{P} S} \right\}$ is closed under binary meets, if S is a strong partition of an element of a complete lattice.