

Entirely defined. Let  $f$  and  $g$  be entirely defined morphisms,  $\text{Dst } f = \text{Src } g$ . Then

$$\begin{aligned} (g \circ f)^\dagger \circ (g \circ f) &= \\ f^\dagger \circ g^\dagger \circ g \circ f &\sqsupseteq \\ f^\dagger \circ 1_{\text{Src } g} \circ f &= \\ f^\dagger \circ 1_{\text{Dst } f} \circ f &= \\ f^\dagger \circ f &\sqsupseteq \\ 1_{\text{Src } f} &= 1_{\text{Src}(g \circ f)}. \end{aligned}$$

So  $g \circ f$  is entirely defined.

That identity morphisms are entirely defined follows from the following:

$$(1_A)^\dagger \circ 1_A = 1_A \circ 1_A = 1_A = 1_{\text{Src } 1_A} \sqsupseteq 1_{\text{Src } 1_A}.$$

□

DEFINITION 260. I will call a *bijective* morphism a morphism which is entirely defined, monovalued, injective, and surjective.

PROPOSITION 261. If a morphism is bijective then it is an isomorphism.

PROOF. Let  $f$  be bijective. Then  $f \circ f^\dagger \sqsubseteq 1_{\text{Dst } f}$ ,  $f^\dagger \circ f \sqsupseteq 1_{\text{Src } f}$ ,  $f^\dagger \circ f \sqsubseteq 1_{\text{Src } f}$ ,  $f \circ f^\dagger \sqsupseteq 1_{\text{Dst } f}$ . Thus  $f \circ f^\dagger = 1_{\text{Dst } f}$  and  $f^\dagger \circ f = 1_{\text{Src } f}$  that is  $f^\dagger$  is an inverse of  $f$ . □

Let Hom-sets be complete lattices.

DEFINITION 262. A morphism  $f$  of a partially ordered category is *metamonovalued* when  $(\prod G) \circ f = \prod_{g \in G} (g \circ f)$  whenever  $G$  is a set of morphisms with a suitable domain and image.

DEFINITION 263. A morphism  $f$  of a partially ordered category is *metainjective* when  $f \circ (\prod G) = \prod_{g \in G} (f \circ g)$  whenever  $G$  is a set of morphisms with a suitable domain and image.

OBVIOUS 264. Metamonovaluedness and metainjectivity are dual to each other.

DEFINITION 265. A morphism  $f$  of a partially ordered category is *metacomplete* when  $f \circ (\sqcup G) = \sqcup_{g \in G} (f \circ g)$  whenever  $G$  is a set of morphisms with a suitable domain and image.

DEFINITION 266. A morphism  $f$  of a partially ordered category is *co-metacomplete* when  $(\sqcup G) \circ f = \sqcup_{g \in G} (g \circ f)$  whenever  $G$  is a set of morphisms with a suitable domain and image.

Let now Hom-sets be meet-semilattices.

DEFINITION 267. A morphism  $f$  of a partially ordered category is *weakly metamonovalued* when  $(g \sqcap h) \circ f = (g \circ f) \sqcap (h \circ f)$  whenever  $g$  and  $h$  are morphisms with a suitable domain and image.

DEFINITION 268. A morphism  $f$  of a partially ordered category is *weakly metainjective* when  $f \circ (g \sqcap h) = (f \circ g) \sqcap (f \circ h)$  whenever  $g$  and  $h$  are morphisms with a suitable domain and image.

Let now Hom-sets be join-semilattices.

DEFINITION 269. A morphism  $f$  of a partially ordered category is *weakly metacomplete* when  $f \circ (g \sqcup h) = (f \circ g) \sqcup (f \circ h)$  whenever  $g$  and  $h$  are morphisms with a suitable domain and image.