I will denote quasicomplement and dual quasicomplement for a specific poset $\mathfrak{A}$ as $a^{*(\mathfrak{A})}$ and $a^{+(\mathfrak{A})}$.

DEFINITION 235. Let $a, b \in \mathfrak{A}$ where $\mathfrak{A}$ is a distributive lattice. *Quasidifference* of a and b is

$$a \setminus^* b = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}.$$

DEFINITION 236. Let $a, b \in \mathfrak{A}$ where $\mathfrak{A}$ is a distributive lattice. *Second quasidifference* of a and b is

$$a \# b = \bigsqcup \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge z \asymp b} \right\}.$$

THEOREM 237. $a \setminus^* b = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge a \sqsubseteq b \sqcup z} \right\}$ where $\mathfrak{A}$ is a distributive lattice and $a, b \in \mathfrak{A}$.

PROOF. Obviously $\left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge a \sqsubseteq b \sqcup z} \right\} \subseteq \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$. Thus $\bigsqcap \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge a \sqsubseteq b \sqcup z} \right\} \sqsupseteq a \setminus^* b$.

Let $z \in \mathfrak{A}$ and $z' = z \sqcap a$.

$a \sqsubseteq b \sqcup z \Rightarrow a \sqsubseteq (b \sqcup z) \sqcap a \Leftrightarrow a \sqsubseteq (b \sqcap a) \sqcup (z \sqcap a) \Leftrightarrow a \sqsubseteq (b \sqcap a) \sqcup z' \Rightarrow a \sqsubseteq b \sqcup z'$ and $a \sqsubseteq b \sqcup z \Leftarrow a \sqsubseteq b \sqcup z'$. Thus $a \sqsubseteq b \sqcup z \Leftrightarrow a \sqsubseteq b \sqcup z'$.

If $z \in \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$ then $a \sqsubseteq b \sqcup z$ and thus

$$z' \in \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge a \sqsubseteq b \sqcup z} \right\}.$$

But $z' \sqsubseteq z$ thus having $\bigsqcap \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge a \sqsubseteq b \sqcup z} \right\} \sqsubseteq \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$. □

REMARK 238. If we drop the requirement that $\mathfrak{A}$ is distributive, two formulas for quasidifference (the definition and the last theorem) fork.

OBVIOUS 239. Dual quasicomplement is the dual of quasicomplement.

OBVIOUS 240.

- Every pseudocomplement is quasicomplement.
- Every dual pseudocomplement is dual quasicomplement.
- Every pseudodifference is quasidifference.

Below we will stick to the more general quasies than pseudos. If needed, one can check that a quasicomplement a^* is a pseudocomplement by the equation $a^* \asymp a$ (and analogously with other quasies).

Next we will express quasidifference through quasicomplement.

PROPOSITION 241.

1°. $a \setminus^* b = a \setminus^* (a \sqcap b)$ for any distributive lattice;
2°. $a \# b = a \# (a \sqcap b)$ for any distributive lattice with least element.

PROOF.

1°. $a \sqsubseteq (a \sqcap b) \sqcup z \Leftrightarrow a \sqsubseteq (a \sqcup z) \sqcap (b \sqcup z) \Leftrightarrow a \sqsubseteq a \sqcup z \wedge a \sqsubseteq b \sqcup z \Leftrightarrow a \sqsubseteq b \sqcup z$. Thus $a \setminus^* (a \sqcap b) = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq (a \sqcap b) \sqcup z} \right\} = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\} = a \setminus^* b$.