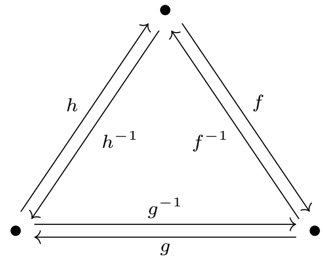
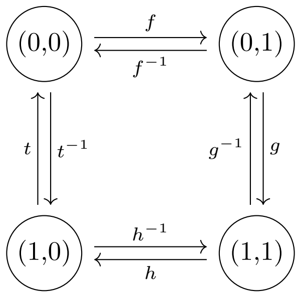
FIGURE 1.



That the diagram is commutative follows from it (because for paths σ, τ we have the paths $\sigma \circ \tau^{-1}$ and $\tau \circ \sigma^{-1}$ being identities). □

LEMMA 194. Let f, g, h, t be isomorphisms. Let $t \circ h \circ g \circ f = 1_{\operatorname{Src} f}$. The diagram at the figure 2 is commutative, every cycle in the diagram is an identity.

FIGURE 2.



PROOF. Assign to every vertex (i, j) of the diagram morphism $W(i, j)$ defined by the table 1.

TABLE 1.

i	j	$W(i, j)$
0	0	$1_{\operatorname{Src} f}$
0	1	f
1	0	t^{-1}
1	1	$g \circ f$

It is easy to verify by induction that the morphism corresponding every cycle in the diagram starting at the vertex $(0, 0)$ and ending with a vertex (x, y) is $W(x, y)$.

Thus the morphism corresponding to every cycle starting at the vertex $(0, 0)$ is identity.

By symmetry, the morphism corresponding to every cycle is identity.

That the diagram is commutative follows from it (because for paths σ, τ we have the paths $\sigma \circ \tau^{-1}$ and $\tau \circ \sigma^{-1}$ being identities). □

2.3. Intro to group theory

DEFINITION 195. A *semigroup* is a pair of a set G and an associative binary operation on G.