REMARK 147. I do not require that $a^*$ is undefined if there are no pseudocomplement of $a$ and likewise for dual pseudocomplement and pseudodifference. In fact below I will define quasicomplement, dual quasicomplement, and quasidifference which generalize pseudo-* counterparts. I will denote $a^*$ the more general case of quasicomplement than of pseudocomplement, and likewise for other notation.

OBVIOUS 148. Dual pseudocomplement is the dual of pseudocomplement.

THEOREM 149. Let $\mathfrak{A}$ be a distributive lattice with least element. Let $a, b \in \mathfrak{A}$. If $a \setminus b$ exists, then $a \setminus^* b$ also exists and $a \setminus^* b = a \setminus b$.

PROOF. Because $\mathfrak{A}$ be a distributive lattice with least element, the definition of $a \setminus b$ is correct.

Let $x = a \setminus b$ and let $S = \left\{ \frac{y \in \mathfrak{A}}{a \sqsubseteq b \sqcup y} \right\}$.

We need to show

1°. $x \in S$;

2°. $y \in S \Rightarrow x \sqsubseteq y$ (for every $y \in \mathfrak{A}$).

Really,

1°. Because $b \sqcup x = a \sqcup b$.

2°.

$$\begin{aligned}
& y \in S \\
\Rightarrow\ & a \sqsubseteq b \sqcup y && \text{(by definition of } S\text{)} \\
\Rightarrow\ & a \sqcup b \sqsubseteq b \sqcup y \\
\Rightarrow\ & x \sqcup b \sqsubseteq b \sqcup y && \text{(since } x \sqcup b = a \sqcup b\text{)} \\
\Rightarrow\ & x \sqcap (x \sqcup b) \sqsubseteq x \sqcap (b \sqcup y) \\
\Rightarrow\ & (x \sqcap x) \sqcup (x \sqcap b) \sqsubseteq (x \sqcap b) \sqcup (x \sqcap y) && \text{(by distributive law)} \\
\Rightarrow\ & x \sqcup \bot \sqsubseteq \bot \sqcup (x \sqcap y) && \text{(since } x \sqcap b = \bot\text{)} \\
\Rightarrow\ & x \sqsubseteq x \sqcap y \\
\Rightarrow\ & x \sqsubseteq y.
\end{aligned}$$

□

DEFINITION 150. *Co-brouwerian lattice* is a lattice for which pseudodifference of any two its elements is defined.

PROPOSITION 151. Every non-empty co-brouwerian lattice $\mathfrak{A}$ has least element.

PROOF. Let $a$ be an arbitrary lattice element. Then

$$a \setminus^* a = \min \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq a \sqcup z} \right\} = \min \mathfrak{A}.$$

So $\min \mathfrak{A}$ exists. □

DEFINITION 152. *Co-Heyting lattice* is co-brouwerian lattice with greatest element.

DEFINITION 153. A *co-frame* is the same as a complete co-brouwerian lattice.

THEOREM 154. For a co-brouwerian lattice $a \sqcup -$ is an upper adjoint of $- \setminus^* a$ for every $a \in \mathfrak{A}$.

PROOF. $g(b) = \min \left\{ \frac{x \in \mathfrak{A}}{a \sqcup x \sqsupseteq b} \right\} = b \setminus^* a$ exists for every $b \in \mathfrak{A}$ and thus is the lower adjoint of $a \sqcup -$. □

COROLLARY 155. $\forall a, x, y \in \mathfrak{A} : (x \setminus^* a \sqsubseteq y \Leftrightarrow x \sqsubseteq a \sqcup y)$ for a co-brouwerian lattice.