DEFINITION 75. I will call b *complementive* to a iff there exists $c \in \mathfrak{A}$ such that $b \sqcap c = \bot$ and $b \sqcup c = a$.

PROPOSITION 76. b is complementive to a iff b is substractive from a and $b \sqsubseteq a$.

PROOF.

$\Leftarrow$. Obvious.
$\Rightarrow$. We deduce $b \sqsubseteq a$ from $b \sqcup c = a$. Thus $a \sqcup b = a = b \sqcup c$.

□

PROPOSITION 77. If b is complementive to a then $(a \setminus b) \sqcup b = a$.

PROOF. Because $b \sqsubseteq a$ by the previous proposition. □

DEFINITION 78. Let $\mathfrak{A}$ be a bounded distributive lattice. The *complement* (denoted $\bar{a}$) of an element $a \in \mathfrak{A}$ is such $b \in \mathfrak{A}$ that $a \sqcap b = \bot$ and $a \sqcup b = \top$.

PROPOSITION 79. If $\mathfrak{A}$ is a bounded distributive lattice then $\bar{a} = \top \setminus a$.

PROOF. $b = \bar{a} \Leftrightarrow b \sqcap a = \bot \wedge b \sqcup a = \top \Leftrightarrow b \sqcap a = \bot \wedge \top \sqcup a = a \sqcup b \Leftrightarrow b = \top \setminus a$. □

COROLLARY 80. If $\mathfrak{A}$ is a bounded distributive lattice then exists no more than one complement of an element $a \in \mathfrak{A}$.

DEFINITION 81. An element of bounded distributive lattice is called *complemented* when its complement exists.

DEFINITION 82. A distributive lattice is a *complemented lattice* iff every its element is complemented.

PROPOSITION 83. For a distributive lattice $(a \setminus b) \setminus c = a \setminus (b \sqcup c)$ if $a \setminus b$ and $(a \setminus b) \setminus c$ are defined.

PROOF. $((a \setminus b) \setminus c) \sqcap c = \bot$; $((a \setminus b) \setminus c) \sqcup c = (a \setminus b) \sqcup c$; $(a \setminus b) \sqcap b = \bot$; $(a \setminus b) \sqcup b = a \sqcup b$.

We need to prove $((a \setminus b) \setminus c) \sqcap (b \sqcup c) = \bot$ and $((a \setminus b) \setminus c) \sqcup (b \sqcup c) = a \sqcup (b \sqcup c)$.

In fact,

$$\begin{aligned}((a \setminus b) \setminus c) \sqcap (b \sqcup c) &= \\ (((a \setminus b) \setminus c) \sqcap b) \sqcup (((a \setminus b) \setminus c) \sqcap c) &= \\ (((a \setminus b) \setminus c) \sqcap b) \sqcup \bot &= \\ ((a \setminus b) \setminus c) \sqcap b &\sqsubseteq \\ (a \setminus b) \sqcap b &= \bot,\end{aligned}$$

so $((a \setminus b) \setminus c) \sqcap (b \sqcup c) = \bot$;

$$\begin{aligned}((a \setminus b) \setminus c) \sqcup (b \sqcup c) &= \\ (((a \setminus b) \setminus c) \sqcup c) \sqcup b &= \\ (a \setminus b) \sqcup c \sqcup b &= \\ ((a \setminus b) \sqcup b) \sqcup c &= \\ a \sqcup b \sqcup c.&\end{aligned}$$

□