

6°. If  $x \in \mathcal{U}$ , then the cardinality of  $x$  is strictly less than the cardinality of  $\mathcal{U}$ .

**1.9.2. Misc.** In this book quantifiers bind tightly. That is  $\forall x \in A : P(x) \wedge Q$  and  $\forall x \in A : P(x) \Rightarrow Q$  should be read  $(\forall x \in A : P(x)) \wedge Q$  and  $(\forall x \in A : P(x)) \Rightarrow Q$  not  $\forall x \in A : (P(x) \wedge Q)$  and  $\forall x \in A : (P(x) \Rightarrow Q)$ .

The set of functions from a set  $A$  to a set  $B$  is denoted as  $B^A$ .

I will often skip parentheses and write  $fx$  instead of  $f(x)$  to denote the result of a function  $f$  acting on the argument  $x$ .

I will denote  $\langle f \rangle^* X = \left\{ \frac{\beta \in \text{im } f}{\exists \alpha \in X : \alpha f \beta} \right\}$  (in other words  $\langle f \rangle^* X$  is the image of a set  $X$  under a function or binary relation  $f$ ) and  $X [f]^* Y \Leftrightarrow \exists x \in X, y \in Y : x f y$  for sets  $X, Y$  and a binary relation  $f$ . (Note that functions are a special case of binary relations.)

By just  $\langle f \rangle^*$  and  $[f]^*$  I will denote the corresponding function and relation on small sets.

OBVIOUS 2. For a function  $f$  we have  $\langle f \rangle^* X = \left\{ \frac{f(x)}{x \in X} \right\}$ .

DEFINITION 3.  $\langle f^{-1} \rangle^* X$  is called the *preimage* of a set  $X$  by a function (or, more generally, a binary relation)  $f$ .

OBVIOUS 4.  $\{\alpha\} [f]^* \{\beta\} \Leftrightarrow \alpha f \beta$  for every  $\alpha$  and  $\beta$ .

$\lambda x \in D : f(x) = \left\{ \frac{(x, f(x))}{x \in D} \right\}$  for a set  $D$  and a form  $f$  depending on the variable  $x$ . In other words,  $\lambda x \in D : f(x)$  is the function which maps elements  $x$  of a set  $D$  into  $f(x)$ .

I will denote source and destination of a morphism  $f$  of any category (See chapter 2 chapter for a definition of a category.) as  $\text{Src } f$  and  $\text{Dst } f$  correspondingly. Note that below defined domain and image of a funcoïd are not the same as its source and destination.

I will denote  $\text{GR}(A, B, f) = f$  for any morphism  $(A, B, f)$  of either **Set** or **Rel**. (See definitions of **Set** and **Rel** below.)

## 1.10. Implicit arguments

Some notation such that  $\perp^{\mathfrak{A}}, \top^{\mathfrak{A}}, \sqcup^{\mathfrak{A}}, \sqcap^{\mathfrak{A}}$  have indexes (in these examples  $\mathfrak{A}$ ).

We will omit these indexes when they can be restored from the context. For example, having a function  $f : \mathfrak{A} \rightarrow \mathfrak{B}$  where  $\mathfrak{A}, \mathfrak{B}$  are posets with least elements, we will concisely denote  $f \perp = \perp$  for  $f \perp^{\mathfrak{A}} = \perp^{\mathfrak{B}}$ . (See below for definitions of these operations.)

NOTE 5. In the above formula  $f \perp = \perp$  we have the first  $\perp$  and the second  $\perp$  denoting different objects.

We will assume (skipping this in actual proofs) that all omitted indexes can be restored from context. (Note that so called dependent type theory computer proof assistants do this like we implicitly.)

## 1.11. Unusual notation

In the chapter “**Common knowledge, part 1**” (which you may skip reading if you are already knowledgeable) some non-standard notation is defined. I summarize here this notation for the case if you choose to skip reading that chapter:

Partial order is denoted as  $\sqsubseteq$ .

Meets and joins are denoted as  $\sqcap, \sqcup, \sqprod, \sqbigsqcup$ .

I call element  $b$  *subtractive* from an elements  $a$  (of a distributive lattice  $\mathfrak{A}$ ) when the difference  $a \setminus b$  exists. I call  $b$  *complementive* to  $a$  when there exists  $c \in \mathfrak{A}$