

1.8. Structure of this book

In the chapter “**Common knowledge, part 1**” some well known definitions and theories are considered. You may skip its reading if you already know it. That chapter contains info about:

- posets;
- lattices and complete lattices;
- Galois connections;
- co-brouwerian lattices;
- a very short intro into category theory;
- a very short introduction to group theory.

Afterward there are my little additions to poset/lattice and category theory.

Afterward there is the theory of filters and filtrators.

Then there is “**Common knowledge, part 2 (topology)**”, which considers briefly:

- metric spaces;
- topological spaces;
- pretopological spaces;
- proximity spaces.

Despite of the name “Common knowledge” this second common knowledge chapter is recommended to be read completely even if you know topology well, because it contains some rare theorems not known to most mathematicians and hard to find in literature.

Then the most interesting thing in this book, the theory of functors, starts.

Afterwards there is the theory of reloids.

Then I show relationships between functors and reloids.

The last I research generalizations of functors, *pointfree functors*, *staroids*, and *multifunctors* and some different kinds of products of morphisms.

1.9. Basic notation

I will denote a set definition like $\left\{ \frac{x \in A}{P(x)} \right\}$ instead of customary $\{x \in A \mid P(x)\}$. I do this because otherwise formulas don't fit horizontally into the available space.

1.9.1. Grothendieck universes. We will work in ZFC with an infinite and uncountable Grothendieck universe.

A Grothendieck universe is just a set big enough to make all usual set theory inside it. For example if \mathcal{U} is a Grothendieck universe, and sets $X, Y \in \mathcal{U}$, then also $X \cup Y \in \mathcal{U}$, $X \cap Y \in \mathcal{U}$, $X \times Y \in \mathcal{U}$, etc.

A set which is a member of a Grothendieck universe is called a *small set* (regarding this Grothendieck universe). We can restrict our consideration to small sets in order to get rid troubles with proper classes.

DEFINITION 1. Grothendieck universe is a set \mathcal{U} such that:

- 1°. If $x \in \mathcal{U}$ and $y \in x$ then $y \in \mathcal{U}$.
- 2°. If $x, y \in \mathcal{U}$ then $\{x, y\} \in \mathcal{U}$.
- 3°. If $x \in \mathcal{U}$ then $\mathcal{P}x \in \mathcal{U}$.
- 4°. If $\left\{ \frac{x_i}{i \in I \in \mathcal{U}} \right\}$ is a family of elements of \mathcal{U} , then $\bigcup_{i \in I} x_i \in \mathcal{U}$.

One can deduce from this also:

- 1°. If $x \in \mathcal{U}$, then $\{x\} \in \mathcal{U}$.
- 2°. If x is a subset of $y \in \mathcal{U}$, then $x \in \mathcal{U}$.
- 3°. If $x, y \in \mathcal{U}$ then the ordered pair $(x, y) = \{\{x, y\}, x\} \in \mathcal{U}$.
- 4°. If $x, y \in \mathcal{U}$ then $x \cup y$ and $x \times y$ are in \mathcal{U} .
- 5°. If $\left\{ \frac{x_i}{i \in I \in \mathcal{U}} \right\}$ is a family of elements of \mathcal{U} , then the product $\prod_{i \in I} x_i \in \mathcal{U}$.