So $L\in\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Rightarrow L\in\mathrm{GR}\, f$. Thus $f\sqsupseteq\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. □

18.5 Finite case

Theorem 18.77. Let n be a finite set.

1. $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}=\downdownarrows\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $\mathfrak{A}$ and $\mathfrak{Z}$ are meet-semilattices and $(\mathfrak{A};\mathfrak{Z})$ is a finitely meet-closed filtrator.
2. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}=\upuparrows\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $(\mathfrak{A};\mathfrak{Z})$ is a primary filtrator over a distributive lattice.

Proof.

1. $L\in\mathrm{GR}\downdownarrows\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow L\in\mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\mathrm{MEET}(\{L_i\mid i\in n\}\cup\{\mathcal{A}\})\Leftrightarrow\bigsqcap^{\mathfrak{A}}_{i\in n}L_i\sqcap\mathcal{A}\neq 0\Leftrightarrow$ (by finiteness)$\Leftrightarrow\bigsqcap^{\mathfrak{Z}}_{i\in n}L_i\sqcap\mathcal{A}\neq 0\Leftrightarrow L\in\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ for every $L\in\prod\mathfrak{Z}$.
2. $L\in\mathrm{GR}\upuparrows\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\mathrm{up}\,L\subseteq\mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\forall K\in\mathrm{up}\,L\colon K\in\mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\forall K\in\mathrm{up}\,L\colon\bigsqcap^{\mathfrak{Z}}_{i\in n}K_i\in\partial\mathcal{A}\Leftrightarrow\forall K\in\mathrm{up}\,L\colon\bigsqcap^{\mathfrak{Z}}_{i\in n}K_i\not\asymp\mathcal{A}\Leftrightarrow$ (by finiteness and theorem 4.44)$\Leftrightarrow\forall K\in\mathrm{up}\,L\colon\bigsqcap^{\mathfrak{A}}_{i\in n}K_i\not\asymp\mathcal{A}\Leftrightarrow\mathcal{A}\in\bigcap\langle\star\rangle\{\bigsqcap^{\mathfrak{A}}_{i\in n}K_i\mid K\in\mathrm{up}\,L\}\Leftrightarrow$ (by the formula for finite meet of filters, theorem 4.111)$\Leftrightarrow\mathcal{A}\in\bigcap\langle\star\rangle\bigsqcap^{\mathfrak{A}}_{i\in n}L_i\Leftrightarrow\forall K\in\bigsqcap^{\mathfrak{A}}_{i\in n}L_i\colon\mathcal{A}\in\star K\Leftrightarrow\forall K\in\bigsqcap^{\mathfrak{A}}_{i\in n}L_i\colon\mathcal{A}\not\asymp K\Leftrightarrow$ (by separability of core, theorem 4.112)$\Leftrightarrow\bigsqcap^{\mathfrak{A}}_{i\in n}L_i\not\asymp\mathcal{A}\Leftrightarrow L\in\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. □

Proposition 18.78. Let $(\mathfrak{A};\mathfrak{Z})$ be a finitely meet closed filtrator. $\downdownarrows\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ and $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ are the same for finite n.

Proof. Because $\bigsqcap^{\mathfrak{Z}}_{i\in\mathrm{dom}\,L}L_i=\bigsqcap^{\mathfrak{A}}_{i\in\mathrm{dom}\,L}L_i$ for finitary L. [FIXME: Are meets defined?] □

18.6 Counter-examples and conjectures

The following example shows that the theorem 18.33 can't be strenghtened:

Example 18.79. For some multifuncoid f on powersets complete in argument k the following formula is false:

$\langle f\rangle_l(L\cup\{(k;\bigsqcup X)\})=\bigsqcup_{x\in X}\langle f\rangle_l(L\cup\{(k;x)\})$ for every $X\in\mathscr{P}\mathfrak{P}_k$, $L\in\prod_{i\in(\mathrm{arity}\,f)\setminus\{k,l\}}\mathfrak{F}_i$.

Proof. Consider multifuncoid $f=\Lambda\,\mathrm{id}^{\mathsf{Strd}}_{\uparrow U[3]}$ where U is an infinite set (of the form $\mathfrak{P}^3$) and $L=(Y)$ where Y is a nonprincipal filter on U.

$\langle f\rangle_0(L\cup\{(k;\bigsqcup X)\})=Y\sqcap\bigsqcup X$;

$\bigsqcup_{x\in X}\langle f\rangle_0(L\cup\{(k;x)\})=\bigsqcup_{x\in X}(Y\sqcap x)$.

It can be $Y\sqcap\bigsqcup X=\bigsqcup_{x\in X}(Y\sqcap x)$ only if Y is principal: Really: $Y\sqcap\bigsqcup X=\bigsqcup_{x\in X}(Y\sqcap x)$ implies $Y\not\asymp\bigsqcup X\Rightarrow\bigsqcup_{x\in X}(Y\sqcap x)\neq 0\Rightarrow\exists x\in X\colon Y\not\asymp x$ and thus Y is principal. But we claimed above that it is nonprincipal. □

Example 18.80. There exists a staroid f and an indexed family X of principal filters (with $\mathrm{arity}\,f=\mathrm{dom}\,X$ and $(\mathrm{form}\,f)_i=\mathrm{Base}(X_i)$ for every $i\in\mathrm{arity}\,f$), such that $f\sqsubseteq\prod^{\mathsf{Strd}}X$ and $Y\sqcap X\notin\mathrm{GR}\,f$ for some $Y\in\mathrm{GR}\,f$.

Remark 18.81. Such examples obviously do not exist if both f is a principal staroid and X and Y are indexed families of principal filters (because for powerset algebras staroidal product is equivalent to Cartesian product). This makes the above example inspired.

Proof. (Monroe Eskew) Let a be any (trivial or nontrivial) ultrafilter on an infinite set U. Let $A,B\in a$ be such that $A\cap B\subset A,B$. In other words, A, B are arbitrary nonempty sets such that $\emptyset\neq A\cap B\subset A,B$ and a be an ultrafilter on $A\cap B$.