$\forall Y \in \operatorname{up} \mathcal{X} \colon \bigsqcap^{\mathfrak{Z}}_{i \in n} Y_i \sqcap \uparrow A \neq 0$ does not hold for $n = \mathbb{N}$, $\mathcal{X}_i = \uparrow(-1/i; 0)$ for $i \in n$, $A = (-\infty; 0)$. To show this, it's enough to prove $\bigsqcap^{\mathfrak{Z}}_{i \in n} Y_i \sqcap \uparrow A \neq 0$ for $Y_i = \uparrow(-1/i; 0)$ but this is obvious since $\bigsqcap^{\mathfrak{Z}}_{i \in n} Y_i = 0$.

On the other hand, $\bigsqcap^{\mathfrak{A}}_{i \in n} \mathcal{X}_i \sqcap \uparrow A \neq 0$ for the same $\mathcal{X}$ and A. □

The above theorems are summarized in the following diagram:

$$\begin{array}{ccc}
\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]} & \sqsupseteq & \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \\
\downarrow \upuparrows \quad \uparrow \downdownarrows & & \downarrow \upuparrows \quad \uparrow \downdownarrows \\
\mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]} & \sqsupseteq & \upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}
\end{array}$$

Remark 18.73. $\sqsubseteq$ on the diagram means inequality which can become strict for some A and n.

18.4.7 Identity staroids represented as meets and joins

Proposition 18.74. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} = \bigsqcap \{\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \mid A \in a\}$ for every filter a on a powerset where the meet may be taken on every of the following posets: anchored relations, staroids.

Proof. That $\mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]}$ for every $A \in a$ is obvious.

Let $f \sqsubseteq \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]}$ for every $A \in a$. $L \in \mathrm{GR}\, f \Rightarrow L \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Rightarrow \forall A \in a \colon \bigsqcap^{\mathfrak{A}}_{i \in n} L_i \not\asymp A \Rightarrow \bigsqcap^{\mathfrak{A}}_{i \in n} L_i \not\asymp a \Rightarrow L \in \mathrm{GR}\, \mathrm{id}^{\mathsf{Strd}}_{a[n]}$. Thus $f \sqsubseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$. □

Proposition 18.75. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \bigsqcup \left\{ \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \mid a \in \operatorname{atoms} \mathcal{A} \right\} = \bigsqcup \{a^n_{\mathsf{Strd}} \mid a \in \operatorname{atoms} \mathcal{A}\}$ where the join may be taken on every of the following posets: anchored relations, staroids, completary staroids, provided that $\mathcal{A}$ is a filter on a set.

Proof. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \operatorname{atoms} \mathcal{A}$ is obvious.

Let $f \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \operatorname{atoms} \mathcal{A}$. Then $\forall L \in \mathrm{GR}\, \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \colon L \in \mathrm{GR}\, f$ that is

$$\forall L \in \operatorname{form} f \colon (\mathrm{MEET}(\{L_i \mid i \in n\} \cup \{a\}) \Rightarrow L \in \mathrm{GR}\, f).$$

But $\exists a \in \operatorname{atoms} \mathcal{A} \colon \mathrm{MEET}(\{L_i \mid i \in n\} \cup \{a\}) \Leftrightarrow \exists a \in \operatorname{atoms} \mathcal{A} \colon \bigsqcap^{\mathfrak{A}}_{i \in n} L_i \not\asymp a \Leftarrow \bigsqcap^{\mathfrak{A}}_{i \in n} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$.

So $L \in \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Rightarrow L \in \mathrm{GR}\, f$. Thus $f \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$.

Then use the fact that $\mathrm{ID}^{\mathsf{Strd}}_{a[n]} = a^n_{\mathsf{Strd}}$. □

Proposition 18.76. $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \bigsqcup \left\{ \mathrm{id}^{\mathsf{Strd}}_{a[n]} \mid a \in \operatorname{atoms} \mathcal{A} \right\}$ where the meet may be taken on every of the following posets: anchored relations, staroids, provided that $\mathcal{A}$ is a filter on a set.

Proof. $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \sqsupseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \operatorname{atoms} \mathcal{A}$ is obvious.

Let $f \sqsupseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \operatorname{atoms} \mathcal{A}$. Then $\forall L \in \mathrm{GR}\, \mathrm{id}^{\mathsf{Strd}}_{a[n]} \colon L \in \mathrm{GR}\, f$ that is

$$\forall L \in \operatorname{form} f \colon \left(\bigsqcap^{\mathfrak{Z}}_{i \in n} L_i \not\asymp a \Rightarrow L \in \mathrm{GR}\, f \right).$$

But $\exists a \in \operatorname{atoms} \mathcal{A} \colon \bigsqcap^{\mathfrak{Z}}_{i \in n} L_i \not\asymp a \Leftarrow \bigsqcap^{\mathfrak{Z}}_{i \in n} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$.