Proof. $\mathcal{L} \in \upuparrows \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \operatorname{up} \mathcal{L} \subseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \operatorname{up} \mathcal{L} \subseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall L \in \operatorname{up} \mathcal{L}: L \in \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall L \in \operatorname{up} \mathcal{L}$: $\bigsqcap_{i \in n} L_i \sqcap a \neq 0^{\mathfrak{F}} \Leftrightarrow \bigcup_{i \in n} \mathcal{L}_i \cup a$ has finite intersection property$\Leftrightarrow$(lemma)$\Leftrightarrow \mathcal{L} \in \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$. □

Proposition 18.61. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every filter a and an index set n.

Proof. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} = \downdownarrows \upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$. □

Proposition 18.62. $\mathrm{id}^{\mathsf{Strd}}_{a[a]} \sqsubset \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]}$ for every nontrivial ultrafilter a.

Proof. Suppose $\mathrm{id}^{\mathsf{Strd}}_{a[a]} = \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]}$. Then $\mathrm{ID}^{\mathsf{Strd}}_{a[a]} = \upuparrows \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{a[a]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{a[a]}$ what contradicts to the above. □

Obvious 18.63. $\mathcal{L} \in \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow a \sqcap \bigsqcap_{i \in n} \mathcal{L}_i \neq 0^{\mathfrak{F}}$ if a is an element of a complete lattice.

Obvious 18.64. $\mathcal{L} \in \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall i \in n: \mathcal{L}_i \sqsupseteq a \Leftrightarrow \forall i \in n: \mathcal{L}_i \not\asymp a$ if a is an ultrafilter on $\mathfrak{A}$.

18.4.6 Identity staroids on principal filters

For principal filter $\uparrow A$ (where A is a set) the above definitions coincide with n-ary identity relation, as formulated in the following propositions:

Proposition 18.65. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$.

Proof. $L \in \operatorname{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \prod L \not\asymp \mathrm{id}_{A[n]} \Leftrightarrow \exists t \in A \forall i \in n: t \in L_i \Leftrightarrow \bigcap_{i \in n} L_i \cap A \neq \emptyset \Leftrightarrow L \in \operatorname{GR} \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$. Thus $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$. □

Corollary 18.66. $\mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$ is a principal staroid.

Question 18.67. Is $\mathrm{ID}^{\mathsf{Strd}}_{A[n]}$ principal for every principal filter A on a set and index set n?

Proposition 18.68. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubseteq \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for every set A.

Proof. $L \in \operatorname{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow L \in \operatorname{GR} \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \Leftrightarrow \uparrow A \not\asymp \bigsqcap^{\mathfrak{A}}_{i \in n} L_i \Leftarrow \uparrow A \not\asymp \bigsqcap^{\mathfrak{Z}}_{i \in n} L_i \Leftrightarrow L \in \downdownarrows \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$. □

Proposition 18.69. $\uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubset \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for some set A and index set n.

Proof. $L \in \operatorname{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \bigsqcap^{\mathfrak{Z}}_{i \in n} L_i \not\asymp \uparrow A$ what is not implied by $\bigsqcap^{\mathfrak{A}}_{i \in n} L_i \not\asymp \uparrow A$ that is $L \in \downdownarrows \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$. (For a counter example take $n = \mathbb{N}$, $L_i = (0; 1/i)$, $A = \mathbb{R}$.) □

Proposition 18.70. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$.

Proof. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]}$ is obvious from the above. □

Proposition 18.71. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubseteq \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$.

Proof. $\mathcal{X} \in \operatorname{GR} \upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \operatorname{up} \mathcal{X} \subseteq \operatorname{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{X}: Y \in \operatorname{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{X}$: $Y \in \mathrm{id}^{\mathsf{Strd}}_{\uparrow A[n]} \Leftrightarrow \forall Y \in \operatorname{up} \mathcal{X}: \bigsqcap^{\mathfrak{Z}}_{i \in n} Y_i \sqcap \uparrow A \neq 0 \Rightarrow \bigsqcap^{\mathfrak{A}}_{i \in n} \mathcal{X}_i \sqcap \uparrow A \neq 0 \Leftrightarrow \mathcal{X} \in \operatorname{GR} \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$. □

Proposition 18.72. $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \sqsubset \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ for some set A.

Proof. We need to prove $\upuparrows \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \neq \mathrm{ID}^{\mathsf{Strd}}_{\uparrow A[n]}$ that is it's enough to prove (see the above proof) that $\forall Y \in \operatorname{up} \mathcal{X}: \bigsqcap^{\mathfrak{Z}}_{i \in n} Y_i \sqcap \uparrow A \neq 0 \nLeftarrow \bigsqcap^{\mathfrak{A}}_{i \in n} \mathcal{X}_i \sqcap \uparrow A \neq 0$. A counter-example follows: