

(1) \Rightarrow (3). $Y \not\prec \langle f \rangle_l (L \cup \{(k; \sqcup X)\}) \Leftrightarrow \sqcup X \not\prec \langle f \rangle_k (L \cup \{(l; Y)\}) \Leftrightarrow \exists x \in X: x \not\prec \langle f \rangle_k (L \cup \{(l; Y)\}) \Leftrightarrow \exists x \in X: Y \not\prec \langle f \rangle_l (L \cup \{(k; x)\}) \Leftrightarrow Y \not\prec \sqcup_{x \in X} \langle f \rangle_l (L \cup \{(k; x)\})$ for every principal Y . \square

18.4 Identity staroids and multifuncoids

18.4.1 Identity relations

Denote $\text{id}_{A[n]} = \{(\lambda i \in n: x) \mid x \in A\} = \{n \times \{x\} \mid x \in A\}$ the n -ary identity relation on a set A (for each index set n).

Proposition 18.34. $\prod X \not\prec \text{id}_{A[n]} \Leftrightarrow \bigcap_{i \in n} X_i \cap A \neq \emptyset$.

Proof. $\prod X \not\prec \text{id}_{A[n]} \Leftrightarrow \exists t \in A: n \times \{t\} \in \prod X \Leftrightarrow \exists t \in A \forall i \in n: t \in X_i \Leftrightarrow \bigcap_{i \in n} X_i \cap A \neq \emptyset$. \square

18.4.2 General definitions of identity staroids

Consider a filtrator $(\mathfrak{A}; \mathfrak{J})$ and $\mathcal{A} \in \mathfrak{A}$.

I will define below *small identity staroids* $\text{id}_{\mathcal{A}[n]}^{\text{Strd}}$ and *big identity staroids* $\text{ID}_{\mathcal{A}[n]}^{\text{Strd}}$. That they are really staroids and even cometary staroids (under certain conditions) is proved below.

Definition 18.35. Consider a filtrator $(\mathfrak{A}; \mathfrak{J})$. Let \mathfrak{J} be a complete lattice. Let $\mathcal{A} \in \mathfrak{A}$, let n be an index set.

form $\text{id}_{\mathcal{A}[n]}^{\text{Strd}} = \mathfrak{J}^n$; $L \in \text{GR id}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \prod_{i \in n}^{\mathfrak{J}} L_i \in \partial \mathcal{A}$.

Obvious 18.36. $X \in \text{GR id}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \forall A \in \text{up } \mathcal{A}: \prod_{i \in n}^{\mathfrak{J}} X_i \cap A \neq \emptyset$ if our filtrator is with separable core.

Definition 18.37. The subset X of a poset \mathfrak{A} has a *nontrivial lower bound* (I denote this predicate as $\text{MEET}(X)$) iff there is nonleast $a \in \mathfrak{A}$ such that $\forall x \in X: a \sqsubseteq x$.

Definition 18.38. Staroid $\text{ID}_{\mathcal{A}[n]}^{\text{Strd}}$ (for any $\mathcal{A} \in \mathfrak{A}$ where \mathfrak{A} is a poset) is defined by the formulas:

form $\text{ID}_{\mathcal{A}[n]}^{\text{Strd}} = \mathfrak{A}^n$; $\mathcal{L} \in \text{GR ID}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \text{MEET}(\{\mathcal{L}_i \mid i \in n\} \cup \{\mathcal{A}\})$.

Obvious 18.39. If \mathfrak{A} is complete lattice, then $\mathcal{L} \in \text{GR ID}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \prod \mathcal{L} \not\prec \mathcal{A}$.

Obvious 18.40. If \mathfrak{A} is complete lattice and a is an atom, then $\mathcal{L} \in \text{GR ID}_{a[n]}^{\text{Strd}} \Leftrightarrow \prod \mathcal{L} \supseteq a$.

Obvious 18.41. If \mathfrak{A} is a complete lattice then there exists a multifuncoid $\Lambda \text{ID}_{\mathcal{A}[n]}^{\text{Strd}}$ such that $\langle \Lambda \text{ID}_{\mathcal{A}[n]}^{\text{Strd}} \rangle_k L = \prod_{i \in n} L_i \cap \mathcal{A}$ for every $k \in n$, $L \in \mathfrak{A}^{n \setminus \{k\}}$.

Proposition 18.42. If $(\mathfrak{A}; \mathfrak{J})$ is a meet-closed filtrator and \mathfrak{J} is a complete lattice and \mathfrak{A} is a meet-semilattice. There exists a multifuncoid $\Lambda \text{id}_{\mathcal{A}[n]}^{\text{Strd}}$ such that $\langle \Lambda \text{id}_{\mathcal{A}[n]}^{\text{Strd}} \rangle_k L = \prod_{i \in n}^{\mathfrak{J}} L_i \cap^{\mathfrak{A}} \mathcal{A}$ for every $k \in n$, $L \in \mathfrak{J}^{n \setminus \{k\}}$.

Proof. We need to prove that $L \cup \{(k; X)\} \in \text{id}_{\mathcal{A}[n]}^{\text{Strd}} \Leftrightarrow \prod_{i \in n}^{\mathfrak{J}} L_i \cap^{\mathfrak{A}} \mathcal{A} \not\prec^{\mathfrak{A}} X$. But

$$\prod_{i \in n}^{\mathfrak{J}} L_i \cap^{\mathfrak{A}} \mathcal{A} \not\prec^{\mathfrak{A}} X \Leftrightarrow \prod_{i \in n} L_i \cap^{\mathfrak{A}} X \not\prec^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \prod_{i \in n} (L \cup \{(k; X)\})_i \not\prec^{\mathfrak{A}} \mathcal{A} \Leftrightarrow L \cup \{(k; X)\} \in \text{id}_{\mathcal{A}[n]}^{\text{Strd}}. \quad \square$$

18.4.3 Identities are staroids

Proposition 18.43. Let \mathfrak{A} be a complete distributive lattice and $\mathcal{A} \in \mathfrak{A}$. Then $\text{ID}_{\mathcal{A}[n]}^{\text{Strd}}$ is a staroid.