Since $x_i$ is non-zero there exist $p$ such that $\langle x_i\rangle p_i$ is non-zero. Take $k\in n$, $p'_i=p_i$ for $i\neq k$ and $p'_k=q$ for an arbitrary value $q$; then (using the staroidal projections from the previous subsection)

$$\langle x_k\rangle q=\operatorname{Pr}_k^{\mathsf{Strd}}\prod_{i\in n}^{\mathsf{Strd}}\langle x_i\rangle p'_i=\operatorname{Pr}_k^{\mathsf{Strd}}\left\langle\prod^{(C)}x\right\rangle\prod^{\mathsf{Strd}}p'.$$

So the value of $x$ can be restored from $\prod^{(C)}x$ by this formula. □

### 17.14.3 Subatomic product

**Proposition 17.203.** Values $x_i$ (for every $i\in\operatorname{dom}x$) can be restored from the value of $\prod^{(A)}x$ provided that $x$ is an indexed family of non-zero funcoids.

**Proof.** Fix $k\in\operatorname{dom}f$. Let for some filters $x$ and $y$

$$a=\begin{cases}1^{\mathfrak{F}(\operatorname{Base}(x))} & \text{if } i\neq k;\\ x & \text{if } i=k\end{cases}\quad\text{and}\quad b=\begin{cases}1^{\mathfrak{F}(\operatorname{Base}(y))} & \text{if } i\neq k;\\ y & \text{if } i=k.\end{cases}$$

Then $x\,[x_k]\,y\Leftrightarrow a_k\,[x_k]\,b_k\Leftrightarrow\forall i\in\operatorname{dom}f\colon a_i\,[x_i]\,b_i\Leftrightarrow\prod^{\mathsf{RLD}}a\left[\prod^{(A)}x\right]\prod^{\mathsf{RLD}}b$. So we have restored $x_k$ from $\prod^{(A)}x$. □

**Definition 17.204.** For every funcoid $f\colon\prod A\rightarrow\prod B$ (where $A$ and $B$ are indexed families of sets) consider the funcoid $\operatorname{Pr}_k^{(A)}f$ defined by the formula

$$X\left[\operatorname{Pr}_k^{(A)}f\right]^{*}Y\Leftrightarrow\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}X & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right).$$

**Proposition 17.205.** $\operatorname{Pr}_k^{(A)}f$ is really a funcoid.

**Proof.** $\neg\left(\emptyset\left[\operatorname{Pr}_k^{(A)}f\right]^{*}Y\right)$ is obvious.

$$\begin{aligned}
&I\cup J\left[\operatorname{Pr}_k^{(A)}f\right]^{*}Y\Leftrightarrow\\
&\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}(I\cup J) & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right)\Leftrightarrow\\
&\prod_{i\in\operatorname{domA}}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}I\sqcup\uparrow^{A_i}J & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right)\Leftrightarrow\\
&\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}I & \text{if } i=k\end{cases}\right)\sqcup\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}J & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right)\Leftrightarrow\\
&\prod_{i\in\operatorname{domA}}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}I & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right)\vee\\
&\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \uparrow^{A_i}J & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \uparrow^{B_i}Y & \text{if } i=k\end{cases}\right)\Leftrightarrow\\
&I\left[\operatorname{Pr}_k^{(A)}f\right]^{*}Y\vee J\left[\operatorname{Pr}_k^{(A)}f\right]^{*}Y.
\end{aligned}$$

The rest follows from symmetry. □

**Proposition 17.206.** For every funcoid $f\colon\prod A\rightarrow\prod B$ (where $A$ and $B$ are indexed families of sets) there exists a funcoid $\operatorname{Pr}_k^{(A)}f$ defined by the formula

$$\mathcal{X}\left[\operatorname{Pr}_k^{(A)}f\right]\mathcal{Y}\Leftrightarrow\prod_{i\in\operatorname{dom}A}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(A_i)} & \text{if } i\neq k;\\ \mathcal{X} & \text{if } i=k\end{cases}\right)[f]\prod_{i\in\operatorname{dom}B}^{\mathsf{RLD}}\left(\begin{cases}1^{\mathfrak{F}(B_i)} & \text{if } i\neq k;\\ \mathcal{Y} & \text{if } i=k\end{cases}\right).$$