$y \not\asymp \prod^{\mathsf{RLD}}_{i\in\operatorname{dom} f} \langle f_i\rangle \Pr^{\mathsf{RLD}}_i x \Leftrightarrow \forall i \in \operatorname{dom} f\colon \Pr^{\mathsf{RLD}}_i y \not\asymp \langle f_i\rangle \Pr^{\mathsf{RLD}}_i x \Leftrightarrow \forall i \in \operatorname{dom} f\colon \Pr^{\mathsf{RLD}}_i x\, [f_i]\, \Pr^{\mathsf{RLD}}_i y \Leftrightarrow x \left[\prod^{(A)} f\right] y \Leftrightarrow y \not\asymp \left\langle \prod^{(A)} f \right\rangle x$.

Thus $\left\langle \prod^{(A)} f \right\rangle x = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom} f} \langle f_i\rangle \Pr^{\mathsf{RLD}}_i x$. □

Corollary 17.194. $\langle f \times^{(A)} g\rangle x = \langle f\rangle(\operatorname{dom} x) \times^{\mathsf{RLD}} \langle g\rangle(\operatorname{im} x)$.

17.14 On products and projections

Conjecture 17.195. For principal funcoids $\prod^{(C)}$ and $\prod^{(A)}$ coincide with the conventional product of binary relations.

17.14.1 Staroidal product

Let f be a staroid, whose form components are boolean lattices.

Definition 17.196. *Staroidal projection* of a staroid f is the filter $\Pr^{\mathsf{Strd}}_k f$ corresponding to the free star

$$(\operatorname{val} f)_k\big(\lambda i \in (\operatorname{arity} f) \setminus \{k\}\colon 1^{(\operatorname{form} f)_i}\big).$$

Proposition 17.197. $\Pr_k\ \operatorname{GR} \prod^{\mathsf{Strd}} x = \star x_k$ if x is an indexed family of proper filters, and $k \in \operatorname{dom} x$.

Proof. $\Pr_k \operatorname{GR} \prod^{\mathsf{Strd}} x = \Pr_k \left\{ L \in \prod_{i\in\operatorname{dom} x} \operatorname{form} x_i \mid \forall i \in \operatorname{dom} x\colon x_i \not\asymp L_i \right\} =$ (used the fact that x_i are proper filters)$= \{ l \in \operatorname{form} x_k \mid x_k \not\asymp l\} = \star x_k$. □

Proposition 17.198. $\Pr^{\mathsf{Strd}}_k \prod^{\mathsf{Strd}} x = x_k$ if x is an indexed family of proper filters, and $k \in \operatorname{dom} x$.

Proof. $\partial\ \Pr^{\mathsf{Strd}}_k \prod^{\mathsf{Strd}} x = \left(\operatorname{val} \prod^{\mathsf{Strd}} x\right)_k \left(\lambda i \in (\operatorname{dom} x) \setminus \{k\}\colon 1^{(\operatorname{form} x)_i}\right) = \left\{ X \in \left(\operatorname{form} \prod^{\mathsf{Strd}} x\right)_k \mid \left(\lambda i \in (\operatorname{dom} x) \setminus \{k\}\colon 1^{(\operatorname{form} x)_i}\right) \cup \{(k; X)\} \in \operatorname{GR} \prod^{\mathsf{Strd}} x \right\} = \left\{ X \in \operatorname{Base} x_k \mid \left(\forall i \in (\operatorname{dom} x) \setminus \{k\}\colon 1^{(\operatorname{form} x)_i} \not\asymp x_i\right) \wedge X \not\asymp x_k \right\} = \{ X \in \operatorname{Base} x_k \mid X \not\asymp x_k\} = \partial x_k$.

Consequently $\Pr^{\mathsf{Strd}}_k \prod^{\mathsf{Strd}} x = x_k$. □

17.14.2 Cross-composition product of pointfree funcoids

Definition 17.199. *Zero* pointfree funcoid from a poset $\mathfrak{A}$ to to a poset $\mathfrak{B}$ is such a pointfree funcoid $\mathfrak{A} \to \mathfrak{B}$ that $\langle f\rangle x$ is a least element of $\mathfrak{B}$ for every $x \in \mathfrak{A}$.

Proposition 17.200. A pointfree funcoid f is zero iff $[f] = \emptyset$.

Proof. Direct implication is obvious.

Let now $[f] = \emptyset$. Then $\langle f\rangle x \asymp y$ for every $x \in \operatorname{Src} f$, $y \in \operatorname{Dst} f$ and thus $\langle f\rangle x \asymp \langle f\rangle x$. It is possible only when $\langle f\rangle x = 0^{\operatorname{Dst} f}$. □

Corollary 17.201. A pointfree funcoid is zero iff its reverse is zero.

Proposition 17.202. Values x_i (for every $i \in \operatorname{dom} x$) can be restored from the value of $\prod^{(C)} x$ provided that x is an indexed family of non-zero pointfree funcoids if $\operatorname{Src} f_i$ (for every $i \in n$) is an atomic lattice and every $\operatorname{Dst} f_i$ is an atomic poset with greatest element.

Proof. $\left\langle \prod^{(C)} x \right\rangle \prod^{\mathsf{Strd}} p = \prod^{\mathsf{Strd}}_{i\in n} \langle x_i\rangle p_i$ by theorem 17.154.