Proof. If $S=\emptyset$ then $\prod\left\{\prod^{\mathsf{RLD}} a \mid a\in S\right\}=\prod\emptyset=1^{\mathsf{RLD}(\mathfrak{A})}$ and $\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S=\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\emptyset=\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\emptyset=\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}1^{\mathfrak{F}(\mathfrak{A}_i)}=1^{\mathsf{RLD}(\mathfrak{A})}$, thus $\prod\left\{\prod^{\mathsf{RLD}} a \mid a\in S\right\}=\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S$.

Let $S\neq\emptyset$.

$\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S\sqsubseteq\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\{a_i\}=a_i$ for every $a\in S$ because $a_i\in\operatorname{Pr}_i S$. Thus $\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S\sqsubseteq\prod^{\mathsf{RLD}} a$;

$$\prod\left\{\prod^{\mathsf{RLD}} a \mid a\in S\right\}\sqsupseteq\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S.$$

Now suppose $F\in\operatorname{GR}\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S$. Then there exist $X\in\prod_{i\in\operatorname{dom}\mathfrak{A}}\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S$ such that $F\supseteq\prod X$. It is enough to prove that there exist $a\in S$ such that $F\in\operatorname{GR}\prod^{\mathsf{RLD}} a$. For this it is enough $\prod X\in\operatorname{GR}\prod^{\mathsf{RLD}} a$.

Really, $X_i\in\prod\langle\uparrow^{\mathfrak{F}(\mathfrak{A}_i)}\rangle\operatorname{Pr}_i S$ thus $X_i\in a_i$ for every $a\in S$ because $\operatorname{Pr}_i S\supseteq\{a_i\}$.

Thus $\prod X\in\operatorname{GR}\prod^{\mathsf{RLD}} a$. □

Definition 17.175. I call a multireloid *principal* iff its graph is a principal filter. [TODO: Prove that principal multireloids are the same as multireloid corresponding to a relation.]

Definition 17.176. I call a multireloid *convex* iff it is a join of reloidal products.

Theorem 17.177. $\operatorname{StarComp}(a\sqcup b;f)=\operatorname{StarComp}(a;f)\sqcup\operatorname{StarComp}(b;f)$ for multireloids a, b and an indexed family f of reloids with $\operatorname{Src} f_i=(\operatorname{form} a)_i=(\operatorname{form} b)_i$.

Proof. $\operatorname{GR}(\operatorname{StarComp}(a;f)\sqcup\operatorname{StarComp}(b;f))=\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(A;F)\mid A\in\operatorname{GR} a, F\in\prod_{i\in n}\operatorname{GR} f_i\right\}\sqcup\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} b)}\operatorname{StarComp}(B;F)\mid B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(A;F)\sqcup\uparrow^{\mathsf{RLD}(\operatorname{form} b)}\operatorname{StarComp}(B;F)\mid A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}(\operatorname{StarComp}(A;F)\cup\operatorname{StarComp}(B;F))\mid A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}(\operatorname{StarComp}(A\cup B;F))\mid A\in\operatorname{GR} a, B\in\operatorname{GR} b, F\in\prod_{i\in n}\operatorname{GR} f_i\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\operatorname{form} a)}\operatorname{StarComp}(C;F)\mid C\in\operatorname{GR}(a\sqcup b), F\in\prod_{i\in n}\operatorname{GR} f_i\right\}=\operatorname{GR}\operatorname{StarComp}(a\sqcup b;f)$. □

Conjecture 17.178. $f\sqsubseteq\prod^{\mathsf{RLD}} a\Leftrightarrow\forall i\in\operatorname{arity} f:\operatorname{Pr}^{\mathsf{RLD}}_i f\sqsubseteq a_i$ for every multireloid f and $a_i\in\mathfrak{F}((\operatorname{form} f)_i)$ for every $i\in\operatorname{arity} f$.

17.12.1 Starred reloidal product

Tychonoff product of topological spaces inspired me the following definition, which seems possibly useful just like Tychonoff product:

Definition 17.179. Let a be an n-indexed (n is an arbitrary index set) family of filters on sets. $\prod^{\mathsf{RLD}*} a$ (*starred reloidal product*) is the reloid of the form $\prod_{i\in n}\operatorname{Base}(a_i)$ induced by the filter base

$$\left\{\prod_{i\in n}\left(\begin{cases}A_i & \text{if } i\in m\\ \operatorname{Base}(a_i) & \text{if } i\in n\setminus m\end{cases}\right)\mid m \text{ is a finite subset of } n, A\in\prod(a|_m)\right\}.$$

Obvious 17.180. It is really a filter base.

Obvious 17.181. $\prod^{\mathsf{RLD}*} a\sqsupseteq\prod^{\mathsf{RLD}} a$.

Proposition 17.182. $\prod^{\mathsf{RLD}*} a=\prod^{\mathsf{RLD}} a$ if n is finite.

Proof. Take $m=n$ to show that $\prod^{\mathsf{RLD}*} a\sqsubseteq\prod^{\mathsf{RLD}} a$. □