**Proof.** We need to prove:

1. $\operatorname{StarComp}(\operatorname{StarComp}(m; f); g) = \operatorname{StarComp}(m; \lambda i \in n: g_i \circ f_i)$;
2. $\operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m: \operatorname{id}_{\operatorname{Obj}_m i}) = m$;
3. $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow a \not\asymp \operatorname{StarComp}(b; f^{\dagger})$

(the rest is obvious).

Really,

1. $L \in \operatorname{StarComp}(a; f) \Leftrightarrow \exists y \in a \forall i \in n: y_i \mathrel{f_i} L_i$.

   Define the relation $R(f)$ by the formula $x \mathrel{R(f)} y \Leftrightarrow \forall i \in n: x_i \mathrel{f_i} y_i$. Obviously

   $$R(\lambda i \in n: g_i \circ f_i) = R(g) \circ R(f).$$

   $L \in \operatorname{StarComp}(a; f) \Leftrightarrow \exists y \in a: y \mathrel{R(f)} L$.

   $L \in \operatorname{StarComp}(\operatorname{StarComp}(a; f); g) \Leftrightarrow \exists p \in \operatorname{StarComp}(a; f): p \mathrel{R(g)} L \Leftrightarrow \exists p, y \in a: (y \mathrel{R(f)} p \wedge p \mathrel{R(g)} L) \Leftrightarrow \exists y \in a: y (R(g) \circ R(f)) L \Leftrightarrow \exists y \in a: y \mathrel{R(\lambda i \in n: g_i \circ f_i)} L \Leftrightarrow L \in \operatorname{StarComp}(a; \lambda i \in n: g_i \circ f_i)$ because $p \in \operatorname{StarComp}(a; f) \Leftrightarrow \exists y \in a: y \mathrel{R(f)} p$.
2. Obvious.
3. It follows from the proposition above. □

**Obvious 17.143.** $\operatorname{StarComp}(a \cup b; f) = \operatorname{StarComp}(a; f) \cup \operatorname{StarComp}(b; f)$ for $n$-ary relations $a$, $b$ and an $n$-indexed family $f$ of binary relations.

**Theorem 17.144.** $\left\langle \prod^{(C)} f \right\rangle \prod a = \prod_{i \in n} \langle f_i \rangle a_i$ for every family $f = f_{i \in n}$ of binary relations and $a = a_{i \in n}$ where $a_i$ is a small set (for each $i \in n$).

**Proof.** $L \in \left\langle \prod^{(C)} f \right\rangle \prod a \Leftrightarrow L \in \operatorname{StarComp}(\prod a; f) \Leftrightarrow \exists y \in \prod a \forall i \in n: y_i \mathrel{f_i} L_i \Leftrightarrow \exists y \in \prod a \forall i \in n: \{y_i\} \not\asymp \langle f_i^{-1} \rangle \{L_i\} \Leftrightarrow \forall i \in n \exists y \in a_i: \{y\} \not\asymp \langle f_i^{-1} \rangle \{L_i\} \Leftrightarrow \forall i \in n: a_i \not\asymp \langle f_i^{-1} \rangle \{L_i\} \Leftrightarrow \forall i \in n: \{L_i\} \not\asymp \langle f_i \rangle a_i \Leftrightarrow \forall i \in n: L_i \in \langle f_i \rangle a_i \Leftrightarrow L \in \prod_{i \in n} \langle f_i \rangle a_i$. □

### 17.11.4 Star composition of Rel-morphisms

Define *star composition* for an $n$-ary anchored relation $a$ and an $n$-indexed family $f$ of **Rel**-morphisms as an $n$-ary anchored relation complying with the formulas:

$$\operatorname{Obj}_{\operatorname{StarComp}(a; f)} = \lambda i \in \operatorname{arity} a: \operatorname{Dst} f_i;$$
$$\operatorname{arity} \operatorname{StarComp}(a; f) = \operatorname{arity} a;$$
$$L \in \operatorname{GR} \operatorname{StarComp}(a; f) \Leftrightarrow L \in \operatorname{StarComp}(\operatorname{GR} a; \operatorname{GR} \circ f).$$

(Here I denote $\operatorname{GR}(A; B; f) = f$ for every **Rel**-morphism $f$.)

**Proposition 17.145.** $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow \exists x \in a, y \in b \forall j \in n: x_j \mathrel{f_j} y_j$.

**Proof.** From the previous section. □

**Theorem 17.146.** Relations with above defined compositions form a quasi-invertible category with star-morphisms.

**Proof.** We need to prove:

1. $\operatorname{StarComp}(\operatorname{StarComp}(m; f); g) = \operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m: g_i \circ f_i)$;
2. $\operatorname{StarComp}(m; \lambda i \in \operatorname{arity} m: \operatorname{id}_{\operatorname{Obj}_m i}) = m$;
3. $b \not\asymp \operatorname{StarComp}(a; f) \Leftrightarrow a \not\asymp \operatorname{StarComp}(b; f^{\dagger})$

(the rest is obvious).

It follows from the previous section. □