Thus $\langle f\rangle_j(L\cup\{(i;X\sqcup Y)\})=\langle f\rangle_j(L\cup\{(i;X)\})\sqcup\langle f\rangle_j(L\cup\{(i;Y)\})$. □

Let us consider the filtrator $\left(\prod_{i\in\operatorname{arity} f}\mathfrak{F}((\operatorname{form} f)_i);\prod_{i\in\operatorname{arity} f}(\operatorname{form} f)_i\right)$.

Theorem 17.84. Let $(\mathfrak{A}_i;\mathfrak{Z}_i)$ be a family of join-closed down-aligned filtrators whose both base and core are join-semilattices. Let f be a staroid of the form $\mathfrak{Z}$. Then $\upuparrows f$ is a staroid of the form $\mathfrak{A}$.

Proof. First prove that $\upuparrows f$ is a prestaroid. We need to prove that $0\notin(\operatorname{GR}\upuparrows f)_i$ (that is $\operatorname{up}0\nsubseteq(\operatorname{GR} f)_i$ that is $0\notin(\operatorname{GR} f)_i$ what is true by the theorem conditions) and that for every $\mathcal{X}$, $\mathcal{Y}\in\mathfrak{A}_i$ and $\mathcal{L}\in\prod_{i\in(\operatorname{arity} f)\setminus\{i\}}\mathfrak{A}_i$ where $i\in\operatorname{arity} f$

$$\mathcal{L}\cup\{(i;\mathcal{X}\sqcup\mathcal{Y})\}\in\operatorname{GR}\upuparrows f\Leftrightarrow\mathcal{L}\cup\{(i;\mathcal{X})\}\in\operatorname{GR}\upuparrows f\vee\mathcal{L}\cup\{(i;\mathcal{Y})\}\in\operatorname{GR}\upuparrows f.$$

The reverse implication is obvious. Let $\mathcal{L}\cup\{(i;\mathcal{X}\sqcup\mathcal{Y})\}\in\operatorname{GR}\upuparrows f$. Then for every $L\in\mathcal{L}$ and $X\in\mathcal{X}$, $Y\in\mathcal{Y}$ we have and $X\sqcup^{\mathfrak{Z}_i}Y\sqsupseteq\mathcal{X}\sqcup^{\mathfrak{A}_i}\mathcal{Y}$ thus $L\cup\{(i;X\sqcup^{\mathfrak{Z}_i}Y)\}\in\operatorname{GR} f$ and thus

$$L\cup\{(i;X)\}\in\operatorname{GR} f\vee L\cup\{(i;Y)\}\in\operatorname{GR} f$$

consequently $\mathcal{L}\cup\{(i;\mathcal{X})\}\in\operatorname{GR}\upuparrows f\vee\mathcal{L}\cup\{(i;\mathcal{Y})\}\in\operatorname{GR}\upuparrows f$.

It is left to prove that $\upuparrows f$ is an upper set, but this is obvious. □

There is a conjecture similar to the above theorems:

Conjecture 17.85. $L\in\upuparrows[f]\Rightarrow\upuparrows[f]\cap\prod_{i\in\operatorname{dom}\mathfrak{A}}\operatorname{atoms} L_i\neq\emptyset$ for every multifuncoid f for the filtrator $(\mathfrak{F}^n;\mathfrak{P}^n)$.

Conjecture 17.86. Let $\mho$ be a set, $\mathfrak{F}$ be the set of filters on $\mho$, $\mathfrak{P}$ be the set of principal filters on $\mho$, let n be an index set. Consider the filtrator $(\mathfrak{F}^n;\mathfrak{P}^n)$. Then if f is a completary staroid of the form $\mathfrak{P}^n$, then $\upuparrows f$ is a completary staroid of the form $\mathfrak{F}^n$.

Obvious 17.87. $(\bigsqcup F)K=\bigsqcup_{f\in F} fK$ for every set F of premultifuncoid sketches of the same form $\mathfrak{A}$ and $K\in\prod\mathfrak{A}$ whenever every $\bigsqcup_{f\in F} fK$ is defined.

17.7 Join of multifuncoids

Premultifuncoid sketches are ordered by the formula $f\sqsubseteq g\Leftrightarrow\langle f\rangle\sqsubseteq\langle g\rangle$ where $\sqsubseteq$ in the right part of this formula is the product order. I will denote $\sqcap$, $\sqcup$, $\prod$, $\bigsqcup$ (without an index) the order poset operations on the poset of premultifuncoid sketches.

Remark 17.88. To describe this, the definition of product order is used twice. Let f and g be premultifuncoid sketches of the same form $\mathfrak{A}$

$$\langle f\rangle\sqsubseteq\langle g\rangle\Leftrightarrow\forall i\in\operatorname{dom}\mathfrak{A}:\langle f\rangle_i\sqsubseteq\langle g\rangle_i\quad\text{and}\quad\langle f\rangle_i\sqsubseteq\langle g\rangle_i\Leftrightarrow\forall L\in\prod\mathfrak{Z}|_{(\operatorname{dom}\mathfrak{A})\setminus\{i\}}:\langle f\rangle_i L\sqsubseteq\langle g\rangle_i L.$$

Theorem 17.89. $f\sqcup^{\mathsf{pFCD}(\mathfrak{A})}g=f\sqcup g$ for every premultifuncoids f and g for the same indexed family of starrish join-semilattices filtrators.

Proof. $\alpha_i x\overset{\text{def}}{=}f_i x\sqcup g_i x$. It is enough to prove that α is a premultifuncoid.

We need to prove:

$$L_i\not\asymp\alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_j\not\asymp\alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.$$

Really, $L_i\not\asymp\alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_i\not\asymp f_i L|_{(\operatorname{dom} L)\setminus\{i\}}\sqcup g_i L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_i\not\asymp f_i L|_{(\operatorname{dom} L)\setminus\{i\}}\vee L_i\not\asymp g_i L|_{(\operatorname{dom} L)\setminus\{i\}}\Leftrightarrow L_j\not\asymp f_j L|_{(\operatorname{dom} L)\setminus\{j\}}\vee L_j\not\asymp g_j L|_{(\operatorname{dom} L)\setminus\{j\}}\Leftrightarrow L_j\not\asymp f_j L|_{(\operatorname{dom} L)\setminus\{j\}}\sqcup g_j L|_{(\operatorname{dom} L)\setminus\{j\}}\Leftrightarrow L_j\not\asymp\alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}$. □

Theorem 17.90. $\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})}F=\bigsqcup F$ for every set F of premultifuncoids for the same indexed family of join infinite distributive complete lattices filtrators.