Definition 17.76. I will call a *premultifuncoid* a premultifuncoid sketch such that for every i, $j\in n$ and $L\in\prod\mathfrak{Z}$

$$L_i\not\asymp\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}\Leftrightarrow L_j\not\asymp\alpha_j L|_{(\mathrm{dom}\,L)\setminus\{j\}}. \tag{17.3}$$

Definition 17.77. Let $\mathfrak{A}$ be an indexed family of starrish posets. The prestaroid *corresponding* to a premultifuncoid f is $[f]$ defined by the formula:

$$\mathrm{form}\,[f]=\mathfrak{Z}\quad\text{and}\quad L\in\mathrm{GR}\,[f]\Leftrightarrow L_i\not\asymp\langle f\rangle_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}.$$

Proposition 17.78. The prestaroid corresponding to a premultifuncoid is really a prestaroid.

Proof. By the definition of starrish posets. □

Definition 17.79. I will call a *multifuncoid* a premultifuncoid to which corresponds a staroid.

Definition 17.80. I will call a *completary multifuncoid* a premultifuncoid to which corresponds a completary staroid.

Theorem 17.81. Fix some indexed family $\mathfrak{A}$ of boolean lattices. The the set of premultifuncoids g for the filtrator $(\mathfrak{F}_i;\mathfrak{P}_i)$ bijectively corresponds to set of prestaroids f of form $\mathfrak{P}=\lambda i\in\mathrm{dom}\,\mathfrak{A}$: $\mathfrak{P}_i$ by the formulas:

1. $f=[g]$;
2. $\partial\langle g\rangle_i L=(\mathrm{val}\,f)_i L$ for every $i\in\mathrm{dom}\,\mathfrak{A}$, $L\in\prod\mathfrak{P}|_{\mathrm{dom}\,\mathfrak{A}\setminus\{i\}}$.

Proof. Let f be a prestaroid of the form $\mathfrak{P}$. If α is defined by the formula $\alpha_i L=\langle f\rangle_i L$ then $\partial\alpha_i L=(\mathrm{val}\,f)_i L$. Then

$$L_i\not\asymp\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}\Leftrightarrow L\in f\Leftrightarrow L_j\not\asymp\alpha_j L|_{(\mathrm{dom}\,L)\setminus\{j\}}.$$

For the prestaroid f' defined by the formula $L\in f'\Leftrightarrow L_i\not\asymp\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}$ we have:

$$L\in f'\Leftrightarrow L_i\in\partial\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}\Leftrightarrow L_i\in(\mathrm{val}\,f)_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}\Leftrightarrow L\in f;$$

thus $f'=f$.

Let now α be an indexed family of functions $\alpha_i\in\mathfrak{F}(\mathfrak{Z}_i)^{(\mathrm{dom}\,\mathfrak{Z})\setminus\{i\}}$ conforming to the formula (17.3). Let relation f between posets be defined by the formula $L\in f\Leftrightarrow L_i\not\asymp\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}$. Then

$$(\mathrm{val}\,f)_i L=\{K\in\mathfrak{P}_i\mid K\not\asymp\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}\}=\partial\alpha_i L|_{(\mathrm{dom}\,L)\setminus\{i\}}$$

and thus $(\mathrm{val}\,f)_i L$ is a core star that is f is a prestaroid. For the indexed family α' defined by the formula $\alpha'_i L=\langle f\rangle_i L$ we have

$$\partial\alpha'_i L=\partial\langle f\rangle_i L=\{K\in\mathfrak{P}_i\mid K\not\asymp\alpha_i L\}=\partial\alpha_i L;$$

thus $\alpha'=\alpha$ (taking into account that $\mathfrak{P}_i$ is a boolean lattice).

We have shown that these are bijections. □

Definition 17.82. I will denote Λf the premultifuncoid corresponding to a prestaroid f (for an indexed family of boolean lattices) by the above theorem.

Theorem 17.83. Fix some indexed family $\mathfrak{Z}$ of boolean lattices. $\langle f\rangle_j(L\cup\{(i;X\sqcup Y)\})=\langle f\rangle_j(L\cup\{(i;X)\})\sqcup\langle f\rangle_j(L\cup\{(i;Y)\})$ for every premultifuncoid f for the family $(\mathfrak{F}_i;\mathfrak{P}_i)$ of filtrators and $i,j\in\mathrm{arity}\,f$, $i\neq j$, $L\in\prod_{k\in L\setminus\{i,j\}}\mathfrak{Z}_k$, $X,Y\in\mathfrak{A}_i$. [TODO: It also holds for any finite number of arguments.]

Proof. Let $i\in\mathrm{arity}\,f$ and $L\in\prod_{k\in L\setminus\{i,j\}}\mathfrak{Z}_k$. Let $Z\in\mathfrak{Z}_i$.

$Z\not\asymp\langle f\rangle_j(L\cup\{(i;X\sqcup Y)\})\Leftrightarrow L\cup\{(i;X\sqcup Y),(j;Z)\}\in f\Leftrightarrow X\sqcup Y\in(\mathrm{val}\,f)_i(L\cup\{(j;Z)\})\Leftrightarrow X\in(\mathrm{val}\,f)_i(L\cup\{(j;Z)\}\vee Y\in(\mathrm{val}\,f)_i(L\cup\{(j;Z)\})\Leftrightarrow L\cup\{(i;X),(j;Z)\}\in[f]\vee L\cup\{(i;Y),(j;Z)\}\in[f]\Leftrightarrow Z\not\asymp\langle f\rangle_j(L\cup\{(i;X)\})\vee Z\not\asymp\langle f\rangle_j(L\cup\{(i;Y)\})$.