Proof. We will prove only the first as the second is dual.

$\operatorname{up} a = \{c \in \prod \mathfrak{Z} \mid c \sqsupseteq a\} = \{c \in \prod \mathfrak{Z} \mid \forall i \in \operatorname{dom} a : c_i \sqsupseteq a_i\} = \{c \in \prod \mathfrak{Z} \mid \forall i \in \operatorname{dom} a : c_i \in \operatorname{up} a_i\} = \prod_{i \in \operatorname{dom} a} \operatorname{up} a_i$. □

Proposition 17.31. If every $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a filtered complete lattice filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a filtered complete lattice filtrator.

Proof. That $\prod \mathfrak{A}$ is a complete lattice is already proved above. We have for every $a \in \prod \mathfrak{A}$

$\bigsqcap^{\prod \mathfrak{A}} \operatorname{up} a = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \{x_i \mid x \in \operatorname{up} a\} = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \{x \mid x \in \operatorname{up} a_i\} = \lambda i \in \operatorname{dom} \mathfrak{A} : \bigsqcap \operatorname{up} a_i = \lambda i \in \operatorname{dom} \mathfrak{A} : a_i = a$. □

Proposition 17.32. If every $(\mathfrak{A}_{i \in n}; \mathfrak{Z}_{i \in n})$ is a prefiltered complete lattice filtrator with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$), then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a prefiltered complete lattice filtrator.

Proof. Let $a, b \in \prod \mathfrak{A}$ and $a \neq b$. Then there exists $i \in n$ such that $a_i \neq b_i$ and so $\operatorname{up} a_i \neq \operatorname{up} b_i$. Consequently $\prod_{i \in \operatorname{dom} a} \operatorname{up} a_i \neq \prod_{i \in \operatorname{dom} a} \operatorname{up} b_i$ (taken into account that $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$) that is $\operatorname{up} a \neq \operatorname{up} b$. □

Proposition 17.33. Let every $(\mathfrak{A}_{i \in n}; \mathfrak{Z}_{i \in n})$ be a semifiltered filtrator with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$). Then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a semifiltered filtrator. [TODO: Semifiltered is the same as filtered, remove one of the two statements (which one? they are not equivalent having different theorem conditions!)]

Proof. Let every $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be a semifiltered filtrator. Let $\operatorname{up} a \supseteq \operatorname{up} b$ for some $a, b \in \prod \mathfrak{A}$. Then $\prod_{i \in \operatorname{dom} a} \operatorname{up} a_i \supseteq \prod_{i \in \operatorname{dom} a} \operatorname{up} b_i$ and consequently (taking into account that $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$) $\operatorname{up} a_i \supseteq \operatorname{up} b_i$ for every $i \in n$. Then $\forall i \in n : a_i \sqsubseteq b_i$ that is $a \sqsubseteq b$. □

Proposition 17.34. Let $(\mathfrak{A}_i; \mathfrak{Z}_i)$ be filtrators and each $\mathfrak{Z}_i$ be a complete lattice with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$). For $a \in \prod \mathfrak{A}$:

1. $\operatorname{Cor} a = \lambda i \in \operatorname{dom} a : \operatorname{Cor} a_i$;
2. $\operatorname{Cor}' a = \lambda i \in \operatorname{dom} a : \operatorname{Cor}' a_i$.

Proof. We will prove only the first, because the second is dual.

$\operatorname{Cor} a = \bigsqcap^{\prod \mathfrak{Z}} \operatorname{up} a = \lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \{x_i \mid x \in \operatorname{up} a\} = (\operatorname{up} x \neq \emptyset \text{ taken into account}) = \lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \{x \mid x \in \operatorname{up} a_i\} = \lambda i \in \operatorname{dom} a : \bigsqcap^{\mathfrak{Z}_i} \operatorname{up} a_i = \lambda i \in \operatorname{dom} a : \operatorname{Cor} a_i$. □

Proposition 17.35. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a filtrator with (co-)separable core and each $\mathfrak{A}_i$ has a least (greatest) element, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a filtrator with (co-)separable core.

Proof. We will prove only for separable core, as co-separable core is dual.

$x \asymp^{\prod \mathfrak{A}} y \Leftrightarrow$ (used the fact that $\mathfrak{A}_i$ has a least element)$\Leftrightarrow \forall i \in \operatorname{dom} \mathfrak{A} : x_i \asymp^{\mathfrak{A}_i} y_i \Rightarrow \forall i \in \operatorname{dom} \mathfrak{A} \exists X \in \operatorname{up} x_i : X \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \exists X \in \operatorname{up} x \forall i \in \operatorname{dom} \mathfrak{A} : X_i \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \exists X \in \operatorname{up} x : X \asymp^{\prod \mathfrak{A}} y$ for every $x, y \in \prod \mathfrak{A}$. □

Obvious 17.36.

1. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is a down-aligned filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is a down-aligned filtrator.
2. If each $(\mathfrak{A}_i; \mathfrak{Z}_i)$ is an up-aligned filtrator, then $(\prod \mathfrak{A}; \prod \mathfrak{Z})$ is an up-aligned filtrator.

Proposition 17.37. If every b_i is substractive from a_i where a and b are n-indexed families of distributive lattices with least elements (where n is an index set), then $a \setminus b = \lambda i \in n : a_i \setminus b_i$.

Proof. We need to prove $(\lambda i \in n : a_i \setminus b_i) \sqcap b = 0$ and $a \sqcup b = b \sqcup (\lambda i \in n : a_i \setminus b_i)$.

Really, $(\lambda i \in n : a_i \setminus b_i) \sqcap b = \lambda i \in n : (a_i \setminus b_i) \sqcap b_i = 0$ and $b \sqcup (\lambda i \in n : a_i \setminus b_i) = \lambda i \in n : b_i \sqcup (a_i \setminus b_i) = \lambda i \in n : b_i \sqcup a_i = a \sqcup b$. □

Proposition 17.38. If every $\mathfrak{A}_i$ is a distributive lattice, then $a \setminus^* b = \lambda i \in \operatorname{dom} \mathfrak{A} : a_i \setminus^* b_i$ for every $a, b \in \prod \mathfrak{A}$ whenever every $a_i \setminus^* b_i$ is defined.