Theorem 16.27. Let $\nu \sqsubseteq \nu \circ \nu$. If $f|_{\langle \mu \rangle^* \{x\}} \overset{\nu}{\to} \uparrow^{\mathrm{Ob}\, \mu} \{y\}$ then $\mathrm{xlim}_x f = \tau(y)$.

Proof. $\mathrm{im}\, f|_{\langle \mu \rangle^* \{x\}} \sqsubseteq \langle \nu \rangle^* \{y\}$; $\langle f \rangle \langle \mu \rangle^* \{x\} \sqsubseteq \langle \nu \rangle^* \{y\}$;

$$\begin{aligned}
\nu \circ f|_{\langle \mu \rangle^* \{x\}} &\sqsupseteq \\
(\langle \nu \rangle^* \{y\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}) \circ f|_{\langle \mu \rangle^* \{x\}} &= \\
\langle (f|_{\langle \mu \rangle^* \{x\}})^{-1} \rangle \langle \nu \rangle^* \{y\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\} &= \\
\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle \mu \rangle^* \{x\}} \circ f^{-1} \right\rangle \langle \nu \rangle^* \{y\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\} &\sqsupseteq \\
\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle \mu \rangle^* \{x\}} \circ f^{-1} \right\rangle \langle f \rangle \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\} &= \\
\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle \mu \rangle^* \{x\}} \right\rangle \langle f^{-1} \circ f \rangle \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\} &\sqsupseteq \\
\left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle \mu \rangle^* \{x\}} \right\rangle \left\langle \mathrm{id}^{\mathsf{FCD}}_{\langle \mu \rangle^* \{x\}} \right\rangle \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\} &= \\
\langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}.
\end{aligned}$$

On the other hand, $f|_{\langle \mu \rangle^* \{x\}} \sqsubseteq \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}$;

$\nu \circ f|_{\langle \mu \rangle^* \{x\}} \sqsubseteq \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle \langle \nu \rangle^* \{y\} \sqsubseteq \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}$.

So $\nu \circ f|_{\langle \mu \rangle^* \{x\}} = \langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}$.

$\mathrm{xlim}_x f = \{ \nu \circ f|_{\langle \mu \rangle^* \{x\}} \circ \uparrow r \mid r \in G \} = \{ (\langle \mu \rangle^* \{x\} \times^{\mathsf{FCD}} \langle \nu \rangle^* \{y\}) \circ \uparrow r \mid r \in G \} = \tau(y)$. □

Corollary 16.28. If $\lim^{\nu}_{\langle \mu \rangle^* \{x\}} f = y$ then $\mathrm{xlim}_x f = \tau(y)$.

We have injective τ if $\langle \nu \rangle^* \{y_1\} \sqcap \langle \nu \rangle^* \{y_2\} = 0^{\mathfrak{F}(\mathrm{Ob}\, \mu)}$ for every distinct $y_1, y_2 \in \mathrm{Ob}\, \nu$ that is if ν is T_2-separable.