Proof. $f|_{\langle\mu\rangle\mathcal{A}}\overset{\nu}{\to}\langle f\rangle\mathcal{A}\Leftrightarrow$ (by the lemma)$\Leftrightarrow\langle f\circ\mu|_{\mathcal{A}}\rangle\mathcal{A}\sqsubseteq\langle\nu\circ f\rangle\mathcal{A}\Leftarrow f\circ\mu|_{\mathcal{A}}\sqsubseteq\nu\circ f\Leftrightarrow f\in\mathrm{C}(\mu|_{\mathcal{A}};\nu)$. □

Corollary 16.9. Let μ, ν be endofuncoids, $f\in\mathsf{FCD}(\mathrm{Ob}\,\mu;\mathrm{Ob}\,\nu)$, $\mathcal{A}\in\mathfrak{F}(\mathrm{Ob}\,\mu)$, $\mathrm{Src}\,f=\mathrm{Ob}\,\mu$, $\mathrm{Dst}\,f=\mathrm{Ob}\,\nu$. If $f\in\mathrm{C}(\mu;\nu)$ then $f|_{\langle\mu\rangle\mathcal{A}}\overset{\nu}{\to}\langle f\rangle\mathcal{A}$.

Theorem 16.10. Let μ, ν be endofuncoids, $f\in\mathsf{FCD}(\mathrm{Ob}\,\mu;\mathrm{Ob}\,\nu)$, $\mathcal{A}\in\mathfrak{F}(\mathrm{Ob}\,\mu)$ be an ultrafilter, $\mathrm{Src}\,f=\mathrm{Ob}\,\mu$, $\mathrm{Dst}\,f=\mathrm{Ob}\,\nu$. $f\in\mathrm{C}(\mu|_{\mathcal{A}};\nu)$ iff $f|_{\langle\mu\rangle\mathcal{A}}\overset{\nu}{\to}\langle f\rangle\mathcal{A}$.

Proof. $f|_{\langle\mu\rangle\mathcal{A}}\overset{\nu}{\to}\langle f\rangle\mathcal{A}\Leftrightarrow$ (by the lemma)$\Leftrightarrow\langle f\circ\mu|_{\mathcal{A}}\rangle\mathcal{A}\sqsubseteq\langle\nu\circ f\rangle\mathcal{A}\Leftrightarrow$ (used the fact that $\mathcal{A}$ is an ultrafilter)$\Leftrightarrow f\circ\mu|_{\mathcal{A}}\sqsubseteq\nu\circ f|_{\mathcal{A}}\Leftrightarrow f\circ\mu|_{\mathcal{A}}\sqsubseteq\nu\circ f\Leftrightarrow f\in\mathrm{C}(\mu|_{\mathcal{A}};\nu)$. □

16.3 Limit

Definition 16.11. $\lim^{\mu}f=a$ iff $f\overset{\mu}{\to}\uparrow^{\mathrm{Src}\,\mu}\{a\}$ for a T_2-separable funcoid μ and a non-empty funcoid f such that $\mathrm{Dst}\,f=\mathrm{Dst}\,\mu$.

It is defined correctly, that is f has no more than one limit.

Proof. Let $\lim^{\mu}f=a$ and $\lim^{\mu}f=b$. Then $\mathrm{im}\,f\sqsubseteq\langle\mu\rangle^{*}\{a\}$ and $\mathrm{im}\,f\sqsubseteq\langle\mu\rangle^{*}\{b\}$.

Because $f\neq0^{\mathsf{FCD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}$ we have $\mathrm{im}\,f\neq0^{\mathfrak{F}(\mathrm{Dst}\,f)}$; $\langle\mu\rangle^{*}\{a\}\sqcap\langle\mu\rangle^{*}\{b\}\neq0^{\mathfrak{F}(\mathrm{Dst}\,f)}$; $\uparrow^{\mathrm{Src}\,\mu}\{b\}\sqcap\langle\mu^{-1}\rangle\langle\mu\rangle^{*}\{a\}\neq0^{\mathfrak{F}(\mathrm{Src}\,\mu)}$; $\uparrow^{\mathrm{Src}\,\mu}\{b\}\sqcap\langle\mu^{-1}\circ\mu\rangle\uparrow^{\mathrm{Src}\,\mu}\{a\}\neq0^{\mathfrak{F}(\mathrm{Src}\,\mu)}$; $\{a\}\,[\mu^{-1}\circ\mu]^{*}\,\{b\}$. Because μ is T_2-separable we have $a=b$. □

Definition 16.12. $\lim^{\mu}_{\mathcal{B}}f=\lim^{\mu}(f|_{\mathcal{B}})$.

Remark 16.13. We can also in an obvious way define limit of a reloid.

16.4 Generalized limit

[TODO: Refer readers to http://portonmath.tiddlyspace.com/]

16.4.1 Definition

Let μ and ν be endofuncoids. Let G be a transitive permutation group on $\mathrm{Ob}\,\mu$.

For an element $r\in G$ we will denote $\uparrow r=\uparrow^{\mathsf{FCD}(\mathrm{Ob}\,\mu;\mathrm{Ob}\,\mu)}r$.

We require that μ and every $r\in G$ commute, that is

$$\mu\circ\uparrow r=\uparrow r\circ\mu.$$

We require for every $y\in\mathrm{Ob}\,\nu$

$$\nu\sqsupseteq\langle\nu\rangle^{*}\{y\}\times^{\mathsf{FCD}}\langle\nu\rangle^{*}\{y\}. \tag{16.1}$$

Proposition 16.14. Formula (16.1) follows from $\nu\sqsupseteq\nu\circ\nu^{-1}$.

Proof. Let $\nu\sqsupseteq\nu\circ\nu^{-1}$. Then $\langle\nu\rangle^{*}\{y\}\times^{\mathsf{FCD}}\langle\nu\rangle^{*}\{y\}=\langle\nu\rangle\uparrow^{\mathrm{Ob}\,\nu}\{y\}\times^{\mathsf{FCD}}\langle\nu\rangle\uparrow^{\mathrm{Ob}\,\nu}\{y\}=\nu\circ(\uparrow^{\mathrm{Ob}\,\nu}\{y\}\times^{\mathsf{FCD}}\uparrow^{\mathrm{Ob}\,\nu}\{y\})\circ\nu^{-1}=\nu\circ\uparrow^{\mathsf{FCD}(\mathrm{Ob}\,\nu;\mathrm{Ob}\,\nu)}(\{y\}\times\{y\})\circ\nu^{-1}\sqsubseteq\nu\circ\mathrm{id}^{\mathsf{FCD}(\mathrm{Ob}\,\nu)}\circ\nu^{-1}=\nu\circ\nu^{-1}\sqsubseteq\nu$. □

Remark 16.15. The formula (16.1) usually works if ν is a proximity. It does not work if μ is a pretopology or preclosure.

We are going to consider (generalized) limits of arbitrary functions acting from $\mathrm{Ob}\,\mu$ to $\mathrm{Ob}\,\nu$. (The functions in consideration are not required to be continuous.)