Theorem 14.6. For a filter a we have $a \times^{\mathsf{RLD}} a \sqsubseteq \mathrm{id}^{\mathsf{RLD}(\mathrm{Base}(a))}$ only in the case if $a = 0^{\mathfrak{F}(\mathrm{Base}(a))}$ or a is a trivial ultrafilter.

Proof. If $a \times^{\mathsf{RLD}} a \sqsubseteq \mathrm{id}^{\mathsf{RLD}(\mathrm{Base}(a))}$ then there exists $m \in \mathrm{GR}(a \times^{\mathsf{RLD}} a)$ such that $m \subseteq \mathrm{id}_{\mathrm{Base}(a)}$. Consequently there exist $A, B \in \mathrm{GR}\, a$ such that $A \times B \subseteq \mathrm{id}_{\mathrm{Base}(a)}$ what is possible only in the case when $\uparrow^{\mathrm{Base}(a)} A = \uparrow^{\mathrm{Base}(a)} B = a$ is trivial a ultrafilter or the least filter. □

Corollary 14.7. Reloidal product of a non-trivial atomic filter with itself is non-atomic.

Proof. Obviously $(a \times^{\mathsf{RLD}} a) \sqcap \mathrm{id}^{\mathsf{RLD}(\mathrm{Base}(a))} \neq 0^{\mathfrak{F}(\mathrm{Base}(a))}$ and $(a \times^{\mathsf{RLD}} a) \sqcap \mathrm{id}^{\mathsf{RLD}(\mathrm{Base}(a))} \sqsubset a \times^{\mathsf{RLD}} a$. □

Example 14.8. There exist two atomic reloids whose composition is non-atomic and non-empty.

Proof. Let a be a non-trivial ultrafilter on $\mathbb{N}$ and $x \in \mathbb{N}$. Then

$(a \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}}\{x\}) \circ (\uparrow^{\mathbb{N}}\{x\} \times^{\mathsf{RLD}} a) = \bigsqcap \left\{\uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}((A \times \{x\}) \circ (\{x\} \times A)) \mid A \in a\right\} = \bigsqcap \left\{\uparrow^{\mathsf{RLD}(\mathbb{N};\mathbb{N})}(A \times A) \mid A \in a\right\} = a \times^{\mathsf{RLD}} a$

is non-atomic despite of $a \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}}\{x\}$ and $\uparrow^{\mathbb{N}}\{x\} \times^{\mathsf{RLD}} a$ are atomic. □

Example 14.9. There exists non-monovalued atomic reloid.

Proof. From the previous example it follows that the atomic reloid $\uparrow^{\mathbb{N}}\{x\} \times^{\mathsf{RLD}} a$ is not monovalued. □

Example 14.10. Non-convex reloids exist.

Proof. Let a be a non-trivial ultrafilter. Then $\mathrm{id}_a^{\mathsf{RLD}}$ is non-convex. This follows from the fact that only reloidal products which are below $\mathrm{id}^{\mathsf{RLD}(\mathrm{Base}(a))}$ are reloidal products of ultrafilters and $\mathrm{id}_a^{\mathsf{RLD}}$ is not their join. □

Example 14.11. $(\mathsf{RLD})_{\mathrm{in}} f \neq (\mathsf{RLD})_{\mathrm{out}} f$ for a funcoid f.

Proof. Let $f = \mathrm{id}^{\mathsf{FCD}(\mathbb{N})}$. Then $(\mathsf{RLD})_{\mathrm{in}} f = \bigsqcup \left\{a \times^{\mathsf{RLD}} a \mid a \in \mathrm{atoms}^{\mathfrak{F}(\mathbb{N})}\right\}$ and $(\mathsf{RLD})_{\mathrm{out}} f = \mathrm{id}^{\mathsf{RLD}(\mathbb{N})}$. But as have shown above $a \times^{\mathsf{RLD}} a \not\sqsubseteq \mathrm{id}^{\mathsf{RLD}(\mathbb{N})}$ for non-trivial ultrafilter a, and so $(\mathsf{RLD})_{\mathrm{in}} f \not\sqsubseteq (\mathsf{RLD})_{\mathrm{out}} f$. □

Proposition 14.12. $\mathrm{id}^{\mathsf{FCD}(\mathfrak{U})} \sqcap \uparrow^{\mathsf{FCD}(\mathfrak{U};\mathfrak{U})}((\mathfrak{U} \times \mathfrak{U}) \setminus \mathrm{id}_{\mathfrak{U}}) = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \neq 0^{\mathsf{FCD}(\mathfrak{U};\mathfrak{U})}$ for every infinite set $\mathfrak{U}$.

Proof. Note that $\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \rangle \mathcal{X} = \mathcal{X} \sqcap \Omega(\mathfrak{U})$ for every filter $\mathcal{X}$ on $\mathfrak{U}$.

Let $f = \mathrm{id}^{\mathsf{FCD}(\mathfrak{U})}$, $g = \uparrow^{\mathsf{FCD}(\mathfrak{U};\mathfrak{U})}((\mathfrak{U} \times \mathfrak{U}) \setminus \mathrm{id}_{\mathfrak{U}})$.

Let x be a non-trivial ultrafilter on $\mathfrak{U}$. If $X \in x$ then $\operatorname{card} X \geqslant 2$ (In fact, X is infinite but we don't need this.) and consequently $\langle g \rangle^* X = 1^{\mathfrak{F}(\mathfrak{U})}$. Thus $\langle g \rangle x = 1^{\mathfrak{F}(\mathfrak{U})}$. Consequently

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap 1^{\mathfrak{F}(\mathfrak{U})} = x.$$

Also $\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \rangle x = x \sqcap \Omega(\mathfrak{U}) = x$.

Let now x be a trivial ultrafilter. Then $\langle f \rangle x = x$ and $\langle g \rangle x = 1^{\mathfrak{F}(\mathfrak{U})} \setminus x$. So

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap \left(1^{\mathfrak{F}(\mathfrak{U})} \setminus x\right) = 0^{\mathfrak{F}(\mathfrak{U})}.$$

Also $\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \rangle x = x \sqcap \Omega(\mathfrak{U}) = 0^{\mathfrak{F}(\mathfrak{U})}$.

So $\langle f \sqcap g \rangle x = \langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \rangle x$ for every ultrafilter x on $\mathfrak{U}$. Thus $f \sqcap g = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})}$. □

Example 14.13. There exist binary relations f and g such that $\uparrow^{\mathsf{FCD}(A;B)} f \sqcap \uparrow^{\mathsf{FCD}(A;B)} g \neq \uparrow^{\mathsf{FCD}(A;B)}(f \cap g)$ for some sets A, B such that $f, g \subseteq A \times B$.