The last theorem cannot be generalized from atomic filters to arbitrary filters, as it's shown by the following example:

**Example 13.61.** $\mathcal{A}\geqslant_1\mathcal{B}\wedge\mathcal{B}\geqslant_1\mathcal{A}$ but $\mathcal{A}$ is not isomorphic to $\mathcal{B}$ for some filters $\mathcal{A}$ and $\mathcal{B}$.

**Proof.** Consider $\mathcal{A}=\uparrow^{\mathbb{R}}[0;1]$ and $\mathcal{B}=\prod\{\uparrow^{\mathbb{R}}[0;1+\varepsilon)\mid\varepsilon>0\}$. Then the function $f=\lambda x\in\mathbb{R}:x/2$ witnesses both inequalities $\mathcal{A}\geqslant_1\mathcal{B}$ and $\mathcal{B}\geqslant_1\mathcal{A}$. But these filters cannot be isomorphic because only one of them is principal. □

**Lemma 13.62.** Let $f_0$ and $f_1$ be Set-morphisms. Let $f(x;y)=(f_0x;f_1y)$ for a function $f$. Then

$$\left\langle\uparrow^{\mathsf{FCD}(\mathrm{Src}\,f_0\times\mathrm{Src}\,f_1;\mathrm{Dst}\,f_0\times\mathrm{Dst}\,f_1)}f\right\rangle(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})=\langle\uparrow^{\mathsf{FCD}}f_0\rangle\mathcal{A}\times^{\mathsf{RLD}}\langle\uparrow^{\mathsf{FCD}}f_1\rangle\mathcal{B}.$$

**Proof.** $\left\langle\uparrow^{\mathsf{FCD}(\mathrm{Src}\,f_0\times\mathrm{Src}\,f_1;\mathrm{Dst}\,f_0\times\mathrm{Dst}\,f_1)}f\right\rangle(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})=\left\langle\uparrow^{\mathsf{FCD}(\mathrm{Src}\,f_0\times\mathrm{Src}\,f_1;\mathrm{Dst}\,f_0\times\mathrm{Dst}\,f_1)}f\right\rangle\prod\{\uparrow^{\mathrm{Src}\,f_0\times\mathrm{Src}\,f_1}(A\times B)\mid A\in\mathcal{A},B\in\mathcal{B}\}=\prod\{\uparrow^{\mathrm{Dst}\,f_0\times\mathrm{Dst}\,f_1}\langle f\rangle(A\times B)\mid A\in\mathcal{A},B\in\mathcal{B}\}=\prod\{\uparrow^{\mathrm{Dst}\,f_0\times\mathrm{Dst}\,f_1}(\langle f_0\rangle A\times\langle f_1\rangle B)\mid A\in\mathcal{A},B\in\mathcal{B}\}=\prod\{\uparrow^{\mathrm{Dst}\,f_0}\langle f_0\rangle A\times^{\mathsf{RLD}}\uparrow^{\mathrm{Dst}\,f_1}\langle f_1\rangle B\mid A\in\mathcal{A},B\in\mathcal{B}\}=$ (theorem 6.79)$=\prod\{\uparrow^{\mathrm{Dst}\,f_0}\langle f_0\rangle A\mid A\in\mathcal{A}\}\times^{\mathsf{RLD}}\prod\{\uparrow^{\mathrm{Dst}\,f_1}\langle f_1\rangle B\mid A\in\mathcal{B}\}=\langle\uparrow^{\mathsf{FCD}}f_0\rangle\mathcal{A}\times^{\mathsf{RLD}}\langle\uparrow^{\mathsf{FCD}}f_1\rangle\mathcal{B}$. □

**Theorem 13.63.** Let $f$ be a monovalued reloid. Then $\mathrm{GR}\,f$ is isomorphic to the filter $\operatorname{dom}f$.

**Proof.** Let $f$ be a monovalued reloid. There exists a function $F\in\mathrm{GR}\,f$. Consider the bijective function $p=\lambda x\in\operatorname{dom}F:(x;Fx)$.

$\langle p\rangle\operatorname{dom}F=F$ and consequently $\langle p\rangle\operatorname{dom}f=\prod\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}\langle p\rangle\operatorname{dom}K\mid K\in\mathrm{GR}\,f\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}\langle p\rangle\operatorname{dom}(K\cap F)\mid K\in\mathrm{GR}\,f\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}(K\cap F)\mid K\in\mathrm{GR}\,f\right\}=\prod\left\{\uparrow^{\mathsf{RLD}(\mathrm{Src}\,f;\mathrm{Dst}\,f)}K\mid K\in\mathrm{GR}\,f\right\}=\mathrm{GR}\,f$. Thus $p$ witnesses that $\mathrm{GR}\,f$ is isomorphic to the filter $\operatorname{dom}f$. □

**Corollary 13.64.** The graph of a monovalued reloid with atomic domain is atomic.

**Corollary 13.65.** $\mathrm{GR}\,\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}}$ is isomorphic to $\mathcal{A}$ for every filter $\mathcal{A}$.

**Theorem 13.66.** There are atomic filters incomparable by Rudin-Keisler order.

**Proof.** See [13]. □

**Theorem 13.67.** $\geqslant_1$ and $\geqslant_2$ are different relations.

**Proof.** Consider $a$ is an arbitrary non-empty filter. Then $a\geqslant_1 0^{\mathfrak{F}(\mathrm{Base}(a))}$ but not $a\geqslant_2 0^{\mathfrak{F}(\mathrm{Base}(a))}$. □

**Proposition 13.68.** If $a\geqslant_2 b$ where $a$ is an ultrafilter then $b$ is also an ultrafilter.

**Proof.** $b=\langle\uparrow^{\mathsf{FCD}}f\rangle a$ for some $f:\mathrm{Base}(a)\rightarrow\mathrm{Base}(b)$. So $b$ is an ultrafilter since $f$ is monovalued. □

**Corollary 13.69.** If $a\geqslant_1 b$ where $a$ is an ultrafilter then $b$ is also an ultrafilter or $0^{\mathfrak{F}(\mathrm{Base}(a))}$.

**Proof.** $b\sqsubseteq\langle\uparrow^{\mathsf{FCD}}f\rangle a$ for some $f:\mathrm{Base}(a)\rightarrow\mathrm{Base}(b)$. Therefore $b'=\langle\uparrow^{\mathsf{FCD}}f\rangle a$ is an ultrafilter. From this our statement follows. □

**Proposition 13.70.** Principal filters, generated by sets of the same cardinality, are isomorphic.

**Proof.** Let $A$ and $B$ be sets of the same cardinality. Then there are a bijection $f$ from $A$ to $B$. We have $\langle f\rangle A=B$ and thus $A$ and $B$ are isomorphic. □

**Proposition 13.71.** If a filter is isomorphic to a principal filter, then it is also a principal filter induced by a set with the same cardinality.

**Proof.** Let $A$ be a principal filter and $B$ is a filter isomorphic to $A$. Then there are sets $X\in A$ and $Y\in B$ such that there are a bijection $f:X\rightarrow Y$ such that $\langle f\rangle A=B$.