Suppose for the contrary that A is infinite. Then A contains more than one non-zero points y, z ($y \neq z$). Without loss of generality $y < z$. So we have that $(y; z)$ is not of the form $(y; y)$ nor $(0; y)$ nor $(y; 0)$. Therefore $A \times A$ isn't a subset of Γ. □

12.2 Totally bounded endoreloids

The below is a straightforward generalization of the customary definition of totally bounded sets on uniform spaces (it's proved below that for uniform spaces the below definitions are equivalent).

Definition 12.9. An endoreloid f is *α-totally bounded* ($\operatorname{totBound}_\alpha(f)$) if every $E \in \operatorname{xyGR} f$ is α-thick.

Definition 12.10. An endoreloid f is *β-totally bounded* ($\operatorname{totBound}_\beta(f)$) if every $E \in \operatorname{xyGR} f$ is β-thick.

Remark 12.11. We could rewrite the above definitions in a more algebraic way like $\operatorname{xyGR} f \subseteq \operatorname{thick}_\alpha$ (with $\operatorname{thick}_\alpha$ would be defined as a set rather than as a predicate), but we don't really need this simplification.

Proposition 12.12. If an endoreloid is α-totally bounded then it is β-totally bounded.

Proof. Because $\operatorname{thick}_\alpha(E) \Rightarrow \operatorname{thick}_\beta(E)$. □

Proposition 12.13. If an endoreloid f is reflexive and $\operatorname{Ob} f$ is finite then f is both α-totally bounded and β-totally bounded.

Proof. It enough to prove that f is α-totally bounded. Really, every $E \in \operatorname{xyGR} f$ is reflexive. Thus $\{x\} \times \{x\} \subseteq E$ for $x \in \operatorname{Ob} f$ and thus $\{\{x\} \mid x \in \operatorname{Ob} f\}$ is a sought for finite cover of $\operatorname{Ob} f$. □

Obvious 12.14.

- A principal endoreloid induced by a Rel-morphism E is α-totally bounded iff E is α-thick.
- A principal endoreloid induced by a Rel-morphism E is β-totally bounded iff E is β-thick.

Example 12.15. There is a β-totally bounded endoreloid which is not α-totally bounded.

Proof. It follows from the example above and properties of principal endoreloids. □

12.3 Special case of uniform spaces

Definition 12.16. *Uniform space* is essentially the same as symmetric, reflexive and transitive endoreloid.

Exercise 12.1. Prove that it is essentially the same as the standard definition of a uniform space (see Wikipedia or PlanetMath).

Theorem 12.17. Let f be such a endoreloid that $f \circ f^{-1} \sqsubseteq f$. Then f is α-totally bounded iff it is β-totally bounded.

Proof.

⇒. Proved above.

⇐. For every $\varepsilon \in \operatorname{GR} f$ we have that $\langle\varepsilon\rangle\{c_0\}, \ldots, \langle\varepsilon\rangle\{c_n\}$ covers the space. $\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\} \subseteq \varepsilon \circ \varepsilon^{-1}$ because for $x \in \langle\varepsilon\rangle\{c_i\}$ (the same as $c_i \in \langle\varepsilon^{-1}\rangle\{x\}$) we have $\langle\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\}\rangle\{x\} = \langle\varepsilon\rangle\{c_i\} \subseteq \langle\varepsilon\rangle\langle\varepsilon^{-1}\rangle\{x\} = \langle\varepsilon \circ \varepsilon^{-1}\rangle\{x\}$. For every $\varepsilon' \in \operatorname{GR} f$ exists $\varepsilon \in \operatorname{GR} f$ such that $\varepsilon \circ \varepsilon^{-1} \subseteq \varepsilon'$ because $f \circ f^{-1} \sqsubseteq f$. Thus for every ε' we have $\langle\varepsilon\rangle\{c_i\} \times \langle\varepsilon\rangle\{c_i\} \subseteq \varepsilon'$ and so

$$\langle\varepsilon\rangle\{c_0\}, \ldots, \langle\varepsilon\rangle\{c_n\}.$$