Definition 11.24. A filter $\mathcal{A}\in\mathfrak{F}(\operatorname{Ob}\mu)$ is called *connected* regarding an endofuncoid μ when

$$\forall\mathcal{X},\mathcal{Y}\in\mathfrak{F}(\operatorname{Ob}\mu)\setminus\left\{0^{\mathfrak{F}(\operatorname{Ob}\mu)}\right\}:(\mathcal{X}\sqcup\mathcal{Y}=\mathcal{A}\Rightarrow\mathcal{X}\,[\mu]\,\mathcal{Y}).$$

Proposition 11.25. Let A be a set. The filter $\uparrow^{\operatorname{Ob}\mu}A$ is connected regarding an endofuncoid μ iff

$$\forall X,Y\in\mathscr{P}(\operatorname{Ob}\mu)\setminus\{\emptyset\}:(X\cup Y=A\Rightarrow X\,[\mu]^*\,Y).$$

Proof.

⇒. Obvious.

⇐. It follows from co-separability of filters. □

Theorem 11.26. The following are equivalent for every set A and binary relation μ on a set U:

1. A is connected regarding binary relation μ.
2. $\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)}\mu$.
3. $\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)}\mu$.

Proof.

(1)⇔(2). $S^*\left(\uparrow^{\mathsf{RLD}(U;U)}\mu\sqcap(\uparrow^U A\times^{\mathsf{RLD}}\uparrow^U A)\right)=S^*\left(\uparrow^{\mathsf{RLD}(U;U)}(\mu\cap(A\times A))\right)=\uparrow^{\mathsf{RLD}(U;U)}S(\mu\cap(A\times A))$. So $S^*\left(\uparrow^{\mathsf{RLD}(U;U)}\mu\sqcap(\uparrow^U A\times^{\mathsf{RLD}}\uparrow^U A)\right)\sqsupseteq\uparrow^U A\times^{\mathsf{RLD}}\uparrow^U A\Leftrightarrow\uparrow^{\mathsf{RLD}(U;U)}S(\mu\cap(A\times A))\sqsupseteq\uparrow^{\mathsf{RLD}(U;U)}(A\times A)=\uparrow^U A\times^{\mathsf{RLD}}\uparrow^U A$.

(1)⇔(3). It follows from the previous proposition. □

Next is conjectured a statement more strong than the above theorem:

Conjecture 11.27. Let $\mathcal{A}$ be a filter on a set U and F is a binary relation on U.
$\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)}F$ iff $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)}F$.

Obvious 11.28. A filter $\mathcal{A}$ is connected regarding a reloid μ iff it is connected regarding the reloid $\mu\sqcap(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A})$.

Obvious 11.29. A filter $\mathcal{A}$ is connected regarding a funcoid μ iff it is connected regarding the funcoid $\mu\sqcap(\mathcal{A}\times^{\mathsf{FCD}}\mathcal{A})$.

Theorem 11.30. A filter $\mathcal{A}$ is connected regarding a reloid f iff $\mathcal{A}$ is connected regarding every $F\in\langle\uparrow^{\mathsf{RLD}}\rangle\operatorname{xyGR}f$.

Proof.

⇒. Obvious.

⇐. $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}}F$ iff $S(F)=F^0\sqcup F^1\sqcup F^2\sqcup\ldots\in\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A}$.
$S^*(f)=\bigsqcap\{\uparrow^{\mathsf{RLD}}S(F)\mid F\in\operatorname{xyGR}f\}\sqsupseteq\bigsqcap\{\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A}\mid F\in\operatorname{xyGR}f\}=\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A}$. □

Conjecture 11.31. A filter $\mathcal{A}$ is connected regarding a funcoid f iff $\mathcal{A}$ is connected regarding every $F\in\langle\uparrow^{\mathsf{FCD}}\rangle\operatorname{xyGR}f$.

The above conjecture is open even for the case when $\mathcal{A}$ is a principal filter.

Conjecture 11.32. A filter $\mathcal{A}$ is connected regarding a reloid f iff it is connected regarding the funcoid $(\mathsf{FCD})f$.

The above conjecture is true in the special case of principal filters:

Proposition 11.33. A filter $\uparrow^{\operatorname{Ob}\mu}A$ (for a set A) is connected regarding an endoreloid f iff it is connected regarding the endofuncoid $(\mathsf{FCD})f$.