Chapter 11

Connectedness regarding funcoids and reloids

Definition 11.1. I will call *endoreloids* and *endofuncoids* reloids and funcoids with the same source and destination.[TODO: Move above “continuity” chapter.]

11.1 Some lemmas

Lemma 11.2. If $\neg(A\,[f]^\ast\,B) \wedge A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$ then f is closed on $\uparrow^U A$ for a funcoid $f \in \mathsf{FCD}(U;U)$ for every sets U and $A, B \in \mathscr{P}U$.

Proof. Let $A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$. $\neg(A\,[f]^\ast\,B) \Leftrightarrow \uparrow^U B \sqcap \langle f\rangle \uparrow^U A = 0^{\mathfrak{F}(U)} \Rightarrow (\operatorname{dom} f \sqcup \operatorname{im} f) \sqcap \uparrow^U B \sqcap \langle f\rangle^\ast A = 0^{\mathfrak{F}(U)} \Rightarrow ((\operatorname{dom} f \sqcup \operatorname{im} f) \setminus \uparrow^U A) \sqcap \langle f\rangle^\ast A = 0^{\mathfrak{F}(U)} \Leftrightarrow \langle f\rangle^\ast A \sqsubseteq \uparrow^U A$. □

Corollary 11.3. If $\neg(A\,[f]^\ast\,B) \wedge A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$ then f is closed on $\uparrow^U (A \setminus B)$ for a funcoid $f \in \mathsf{FCD}(U;U)$ for every sets U and $A, B \in \mathscr{P}U$.

Proof. Let $\neg(A\,[f]^\ast\,B) \wedge A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$. Then $\neg((A \setminus B)\,[f]^\ast\,B) \wedge (A \setminus B) \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$. □

Lemma 11.4. If $\neg(A\,[f]^\ast\,B) \wedge A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$ then $\neg(A\,[f^n]^\ast\,B)$ for every whole positive n.

Proof. Let $\neg(A\,[f]^\ast\,B) \wedge A \cup B \in \operatorname{dom} f \sqcup \operatorname{im} f$. From the above lemma $\langle f\rangle^\ast A \sqsubseteq \uparrow^U A$. $\uparrow^U B \sqcap \langle f\rangle \uparrow^U A = 0^{\mathfrak{F}(U)}$, consequently $\langle f\rangle^\ast A \sqsubseteq \uparrow^U (A \setminus B)$. Because (by the above corollary) f is closed on $\uparrow^U (A \setminus B)$, then $\langle f\rangle \langle f\rangle \uparrow^U A \sqsubseteq \uparrow^U (A \setminus B)$, $\langle f\rangle \langle f\rangle \langle f\rangle \uparrow^U A \sqsubseteq \uparrow^U (A \setminus B)$, etc. So $\langle f^n\rangle \uparrow^U A \sqsubseteq \uparrow^U (A \setminus B)$, $\uparrow^U B \asymp \langle f^n\rangle \uparrow^U A$, $\neg(A\,[f^n]^\ast\,B)$. □

11.2 Endomorphism series

Definition 11.5. $S_1(\mu) = \mu \sqcup \mu^2 \sqcup \mu^3 \sqcup \ldots$ for an endomorphism μ of a precategory with countable join of morphisms (that is join defined for every countable set of morphisms).

Definition 11.6. $S(\mu) = \mu^0 \sqcup S_1(\mu) = \mu^0 \sqcup \mu \sqcup \mu^2 \sqcup \mu^3 \sqcup \ldots$ where $\mu^0 = 1_{\operatorname{Ob} \mu}$ (identity morphism for the object $\operatorname{Ob} \mu$) where $\operatorname{Ob} \mu$ is the object of endomorphism μ for an endomorphism μ of a category with countable join of morphisms.

I call S_1 and S *endomorphism series*.

We will consider the collection of all binary relations (on a set $\mho$), as well as the collection of all funcoids and the collection of all reloids on a fixed set, as categories with single object $\mho$ and the identity morphisms $\mathrm{id}_\mho$, $\mathrm{id}^{\mathsf{FCD}(\mho)}$, $\mathrm{id}^{\mathsf{RLD}(\mho)}$.

Proposition 11.7. The relation $S(\mu)$ is transitive for the category of binary relations.