Proof. $\uparrow^{\mathsf{FCD}}\{(x;\ y)\} \sqsubseteq (\mathsf{FCD})g \Leftrightarrow \uparrow^{\mathsf{FCD}}\{(x;\ y)\} \not\asymp (\mathsf{FCD})g \Leftrightarrow \{x\}\ [(\mathsf{FCD})g]^*\ \{y\} \Leftrightarrow \uparrow^{\mathsf{RLD}}\{(x;\ y)\} \not\asymp g \Leftrightarrow \uparrow^{\mathsf{RLD}}\{(x;y)\} \sqsubseteq g$. □

Theorem 8.40. $\operatorname{Cor}(\mathsf{FCD})g = (\mathsf{FCD})\operatorname{Cor} g$ for every reloid g.

Proof. $\operatorname{Cor}(\mathsf{FCD})g = \bigsqcup\{\uparrow^{\mathsf{FCD}}\{(x;\ y)\} \mid \uparrow^{\mathsf{FCD}}\{(x;\ y)\} \sqsubseteq (\mathsf{FCD})g\} = \bigsqcup\{\uparrow^{\mathsf{FCD}}\{(x;\ y)\} \mid \uparrow^{\mathsf{RLD}}\{(x;\ y)\} \sqsubseteq g\} = \bigsqcup\{(\mathsf{FCD})\uparrow^{\mathsf{RLD}}\{(x;y)\} \mid \uparrow^{\mathsf{RLD}}\{(x;y)\} \sqsubseteq g\} = (\mathsf{FCD})\bigsqcup\{\uparrow^{\mathsf{RLD}}\{(x;y)\} \mid \uparrow^{\mathsf{RLD}}\{(x;y)\} \sqsubseteq g\} = (\mathsf{FCD})\operatorname{Cor} g$. □

Conjecture 8.41. For every funcoid g

1. $\operatorname{Cor}(\mathsf{RLD})_{\mathrm{in}}g = (\mathsf{RLD})_{\mathrm{in}}\operatorname{Cor} g$;
2. $\operatorname{Cor}(\mathsf{RLD})_{\mathrm{out}}g = (\mathsf{RLD})_{\mathrm{out}}\operatorname{Cor} g$.

8.4 Funcoidal reloids

Definition 8.42. I call *funcoidal* such a reloid ν that

$$\mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp \nu \Rightarrow \exists \mathcal{X}' \in \mathfrak{F}(\operatorname{Base}(\mathcal{X})) \setminus \{0\}, \mathcal{Y}' \in \mathfrak{F}(\operatorname{Base}(\mathcal{Y})) \setminus \{0\}: (\mathcal{X}' \sqsubseteq \mathcal{X} \wedge \mathcal{Y}' \sqsubseteq \mathcal{Y} \wedge \mathcal{X}' \times^{\mathsf{RLD}} \mathcal{Y}' \sqsubseteq \nu)$$

for every $\mathcal{X} \in \mathfrak{F}(\operatorname{Src}\nu)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst}\nu)$.

Proposition 8.43. A reloid ν is funcoidal iff $x \times^{\mathsf{RLD}} y \not\asymp \nu \Rightarrow x \times^{\mathsf{RLD}} y \sqsubseteq \nu$ for every ultrafilters x and y on respective sets.

Proof.

- **⇒.** $x \times^{\mathsf{RLD}} y \not\asymp \nu \Rightarrow \exists \mathcal{X}' \in \operatorname{atoms} x, \mathcal{Y}' \in \operatorname{atoms} y: \mathcal{X}' \times^{\mathsf{RLD}} \mathcal{Y}' \sqsubseteq \nu \Rightarrow x \times^{\mathsf{RLD}} y \sqsubseteq \nu$.
- **⇐.** $\mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp \nu \Rightarrow \exists x \in \operatorname{atoms}\mathcal{X}, y \in \operatorname{atoms}\mathcal{Y}: x \times^{\mathsf{RLD}} y \not\asymp \nu \Rightarrow \exists x \in \operatorname{atoms}\mathcal{X}, y \in \operatorname{atoms}\mathcal{Y}: x \times^{\mathsf{RLD}} y \sqsubseteq \nu \Rightarrow \exists \mathcal{X}' \in \mathfrak{F}(\operatorname{Base}(\mathcal{X})) \setminus \{0\}, \mathcal{Y}' \in \mathfrak{F}(\operatorname{Base}(\mathcal{Y})) \setminus \{0\}: (\mathcal{X}' \sqsubseteq \mathcal{X} \wedge \mathcal{Y}' \sqsubseteq \mathcal{Y} \wedge \mathcal{X}' \times^{\mathsf{RLD}} \mathcal{Y}' \sqsubseteq \nu)$. □

Proposition 8.44. $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f = \bigsqcup\left\{a \times^{\mathsf{RLD}} b \mid a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}\nu)}, b \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst}\nu)}, a \times^{\mathsf{RLD}} b \not\asymp f\right\}$.

Proof. $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f = \bigsqcup\left\{a \times^{\mathsf{RLD}} b \mid a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, b \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}, a \times^{\mathsf{FCD}} b \sqsubseteq (\mathsf{FCD})f\right\} = \bigsqcup\left\{a \times^{\mathsf{RLD}} b \mid a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, b \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}, a\ [(\mathsf{FCD})f]\ b\right\} = \bigsqcup\left\{a \times^{\mathsf{RLD}} b \mid a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, b \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, a \times^{\mathsf{RLD}} b \not\asymp f\right\}$. □

Definition 8.45. I call $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f$ *funcoidization* of a reloid f.

Lemma 8.46. $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f$ is funcoidal for every reloid f.

Proof. $x \times^{\mathsf{RLD}} y \not\asymp (\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})\, f \Rightarrow x \times^{\mathsf{RLD}} y \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f$. □

Theorem 8.47. $(\mathsf{RLD})_{\mathrm{in}}$ is a bijection from $\mathsf{FCD}(A;B)$ to the set of funcoidal reloids from A to B.

Proof. Let $f \in \mathsf{FCD}(A;B)$. Prove that $(\mathsf{RLD})_{\mathrm{in}} f$ is funcoidal.

Really $(\mathsf{RLD})_{\mathrm{in}} f = (\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}} f$ and thus we can use the lemma stating that it is funcoidal.

It remains to prove $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})\, f = f$ for a funcoidal reloid f. ($(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}\, g = g$ for every funcoid g is already proved above.)

$(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})\ f = \bigsqcup\left\{x \times^{\mathsf{RLD}} y \mid x \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}\nu)}, y \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst}\nu)}, x \times^{\mathsf{RLD}} y \not\asymp f\right\} = \bigsqcup\left\{p \in \operatorname{atoms}(x \times^{\mathsf{RLD}} y) \mid x \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}\nu)}, y \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst}\nu)}, x \times^{\mathsf{RLD}} y \not\asymp f\right\} = \bigsqcup\left\{p \in \operatorname{atoms}(x \times^{\mathsf{RLD}} y) \mid x \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}\nu)}, y \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst}\nu)}, x \times^{\mathsf{RLD}} y \sqsubseteq f\right\} = \bigsqcup \operatorname{atoms} f = f$. □

Corollary 8.48. Funcoidal reloids are convex.

Proof. Every $(\mathsf{RLD})_{\mathrm{in}}\, f$ is obviously convex. □