Proof. For every sets $X\in\mathscr{P}(\operatorname{Src} f)$, $Y\in\mathscr{P}(\operatorname{Dst} f)$

$$\begin{aligned}
X\,[(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}f]^*\,Y &\Leftrightarrow\\
\uparrow^{\operatorname{Src} f}X\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst} f}Y\not\asymp(\mathsf{RLD})_{\mathrm{in}}f &\Leftrightarrow\\
\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} f)}(X\times Y)\not\asymp\bigsqcup\left\{a\times^{\mathsf{RLD}}b\mid a\in\operatorname{atoms}^{\mathfrak{F}(A)},b\in\operatorname{atoms}^{\mathfrak{F}(B)},a\times^{\mathsf{FCD}}b\sqsubseteq f\right\} &\Leftrightarrow\quad(*)\\
\exists a\in\operatorname{atoms}^{\mathfrak{F}(A)},b\in\operatorname{atoms}^{\mathfrak{F}(B)}\colon(a\times^{\mathsf{FCD}}b\sqsubseteq f\wedge a\sqsubseteq\uparrow^{\operatorname{Src} f}X\wedge b\sqsubseteq\uparrow^{\operatorname{Dst} f}Y) &\Leftrightarrow\\
X\,[f]^*\,Y.&
\end{aligned}$$

* proposition 4.215.

Thus $(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}f=f$. □

Remark 8.21. The above theorem allows to represent funcoids as reloids.

Obvious 8.22. $(\mathsf{RLD})_{\mathrm{in}}(\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B})=\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}$ for every filters $\mathcal{A}$, $\mathcal{B}$.

Conjecture 8.23. $(\mathsf{RLD})_{\mathrm{out}}\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}=\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}}$ for every filter $\mathcal{A}$.

Exercise 8.1. Prove that generally $(\mathsf{RLD})_{\mathrm{in}}\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\neq\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}}$.

Conjecture 8.24. $\operatorname{dom}(\mathsf{RLD})_{\mathrm{in}}f=\operatorname{dom} f$ and $\operatorname{im}(\mathsf{RLD})_{\mathrm{in}}f=\operatorname{im} f$ for every funcoid f. [TODO: easy using products of ultrafilters?]

Proposition 8.25. $\operatorname{dom}(f|_{\mathcal{A}})=\mathcal{A}\sqcap\operatorname{dom} f$ for every reloid f and filter $\mathcal{A}\in\mathfrak{F}(\operatorname{Src} f)$.

Proof. $\operatorname{dom}(f|_{\mathcal{A}})=\operatorname{dom}(\mathsf{FCD})(f|_{\mathcal{A}})=\operatorname{dom}((\mathsf{FCD})f)|_{\mathcal{A}}=\mathcal{A}\sqcap\operatorname{dom}(\mathsf{FCD})f=\mathcal{A}\sqcap\operatorname{dom} f$. □

Theorem 8.26. For every composable reloids f, g:

1. If $\operatorname{im} f\sqsupseteq\operatorname{dom} g$ then $\operatorname{im}(g\circ f)=\operatorname{im} g$.
2. If $\operatorname{im} f\sqsubseteq\operatorname{dom} g$ then $\operatorname{dom}(g\circ f)=\operatorname{dom} g$.

Proof.

1. $\operatorname{im}(g\circ f)=\operatorname{im}(\mathsf{FCD})(g\circ f)=\operatorname{im}((\mathsf{FCD})g\circ(\mathsf{FCD})f)=\operatorname{im}(\mathsf{FCD})g=\operatorname{im} g$.
2. Similar. □

Conjecture 8.27. $(\mathsf{RLD})_{\mathrm{in}}(g\circ f)=(\mathsf{RLD})_{\mathrm{in}}g\circ(\mathsf{RLD})_{\mathrm{in}}f$ for every composable funcoids f and g. [TODO: Solved.]

Theorem 8.28. $a\times^{\mathsf{RLD}}b\sqsubseteq(\mathsf{RLD})_{\mathrm{in}}f\Leftrightarrow a\times^{\mathsf{FCD}}b\sqsubseteq f$ for every funcoid f and $a\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}$, $b\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}$. [TODO: Move to "funcoidal reloids" section?]

Proof. $a\times^{\mathsf{FCD}}b\sqsubseteq f\Rightarrow a\times^{\mathsf{RLD}}b\sqsubseteq(\mathsf{RLD})_{\mathrm{in}}f$ is obvious.

$a\times^{\mathsf{RLD}}b\sqsubseteq(\mathsf{RLD})_{\mathrm{in}}f\Rightarrow a\times^{\mathsf{RLD}}b\not\asymp(\mathsf{RLD})_{\mathrm{in}}f\Rightarrow a\,[(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}f]\,b\Rightarrow a\,[f]\,b\Rightarrow a\times^{\mathsf{FCD}}b\sqsubseteq f$. □

Conjecture 8.29. If $\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}\sqsubseteq(\mathsf{RLD})_{\mathrm{in}}f$ then $\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B}\sqsubseteq f$ for every funcoid f and $\mathcal{A}\in\mathfrak{F}(\operatorname{Src} f)$, $\mathcal{B}\in\mathfrak{F}(\operatorname{Dst} f)$.

Theorem 8.30. $\operatorname{GR}(\mathsf{FCD})g\sqsupseteq\operatorname{GR} g$ for every reloid g.

Proof. Let $K\in\operatorname{GR} g$. Then for every sets $X\in\mathscr{P}\operatorname{Src} g$, $Y\in\mathscr{P}\operatorname{Dst} g$

$X\,[K]\,Y\Leftrightarrow X\,[\uparrow^{\mathsf{FCD}}K]^*\,Y\Leftrightarrow X\,[(\mathsf{FCD})\uparrow^{\mathsf{RLD}}K]^*\,Y\Leftarrow X\,[(\mathsf{FCD})g]^*\,Y$.

Thus $\uparrow^{\mathsf{FCD}}K\sqsupseteq(\mathsf{FCD})g$ that is $K\in\operatorname{GR}(\mathsf{FCD})g$. □

Theorem 8.31. $g\circ(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})\circ f=\langle(\mathsf{FCD})f^{-1}\rangle\mathcal{A}\times^{\mathsf{RLD}}\langle(\mathsf{FCD})g\rangle\mathcal{B}$ for every reloids f, g and filters $\mathcal{A}\in\mathfrak{F}(\operatorname{Dst} f)$, $\mathcal{B}\in\mathfrak{F}(\operatorname{Src} g)$. [TODO: Similar proposition for funcoids?]