3. A reloid f is a both monovalued and injective iff there exists an injection (a monovalued and injective binary relation = injective function) $F \in \mathrm{GR}\, f$.

Proof. The reverse implications are obvious. Let's prove the direct implications:

1. Let f be a monovalued reloid. Then $f \circ f^{-1} \sqsubseteq \mathrm{id}^{\mathsf{RLD}(\mathrm{Dst}\, f)}$. So there exists

$$h \in \mathrm{GR}(f \circ f^{-1}) = \mathrm{GR} \bigsqcap \left\{ \uparrow^{\mathsf{RLD}(\mathrm{Dst}\, f; \mathrm{Dst}\, f)}(F \circ F^{-1}) \mid F \in \mathrm{GR}\, f \right\}$$

   such that $\uparrow^{\mathsf{RLD}(\mathrm{Dst}\, f; \mathrm{Dst}\, f)} h \sqsubseteq \mathrm{id}^{\mathsf{RLD}(\mathrm{Dst}\, f)}$. It's simple to show that $\{F \circ F^{-1} \mid F \in \mathrm{GR}\, f\}$ is a filter base. Consequently there exists $F \in \mathrm{GR}\, f$ such that $F \circ F^{-1} \subseteq \mathrm{id}_{\mathrm{Dst}\, f}$ that is F is a function.

2. Similar.

3. Let f be a both monovalued and injective reloid. Then by proved above there exist F, $G \in \mathrm{GR}\, f$ such that F is monovalued and G is injective. Thus $F \cap G \in \mathrm{GR}\, f$ is both monovalued and injective. □

Conjecture 7.43. A reloid f is monovalued iff

$$\forall g \in \mathsf{RLD}(\mathrm{Src}\, f; \mathrm{Dst}\, f) \colon (g \sqsubseteq f \Rightarrow \exists \mathcal{A} \in \mathfrak{F}(\mathrm{Src}\, f) \colon g = f|_{\mathcal{A}}).$$

7.7 Complete reloids and completion of reloids

Definition 7.44. A *complete* reloid is a reloid representable as a join of reloidal products $\uparrow^A\{\alpha\} \times^{\mathsf{RLD}} b$ where $\alpha \in A$ and b is an ultrafilter on B for some sets A and B.

Definition 7.45. A *co-complete* reloid is a reloid representable as a join of reloidal products $a \times^{\mathsf{RLD}} \uparrow^B\{\beta\}$ where $\beta \in B$ and a is an ultrafilter on A for some sets A and B.

I will denote the sets of complete and co-complete reloids correspondingly as $\mathrm{Compl}\mathsf{RLD}$ and $\mathrm{CoCompl}\mathsf{RLD}$.

Obvious 7.46. Complete and co-complete are dual.

Theorem 7.47.

1. A reloid f is complete iff there exists a function $G \colon \mathrm{Src}\, f \to \mathfrak{F}(\mathrm{Dst}\, f)$ such that

$$f = \bigsqcup \{\uparrow^{\mathrm{Src}\, f}\{\alpha\} \times^{\mathsf{RLD}} G(\alpha) \mid \alpha \in \mathrm{Src}\, f\}. \tag{7.1}$$

2. A reloid f is co-complete iff there exists a function $G \colon \mathrm{Dst}\, f \to \mathfrak{F}(\mathrm{Src}\, f)$ such that

$$f = \bigsqcup \{G(\alpha) \times^{\mathsf{RLD}} \uparrow^{\mathrm{Dst}\, f}\{\alpha\} \mid \alpha \in \mathrm{Dst}\, f\}.$$

Proof. We will prove only the first as the second is symmetric.

⇒. Let f be complete. Then take

$$G(\alpha) = \bigsqcup \left\{ b \in \mathrm{atoms}^{\mathfrak{F}(\mathrm{Dst}\, f)} \mid \uparrow^{\mathrm{Src}\, f}\{\alpha\} \times^{\mathsf{RLD}} b \sqsubseteq f \right\}$$

and we have (7.1) obviously.

⇐. Let (7.1) hold. Then $G(\alpha) = \bigsqcup \mathrm{atoms}\, G(\alpha)$ and thus

$$f = \bigsqcup \{\uparrow^{\mathrm{Src}\, f}\{\alpha\} \times^{\mathsf{RLD}} b \mid \alpha \in \mathrm{Src}\, f, b \in \mathrm{atoms}\, G(\alpha)\}$$

and so f is complete. □

Obvious 7.48. Complete and co-complete reloids are convex.