Lemma 7.38. $\lambda \mathcal{B} \in \mathfrak{F}(B): 1^{\mathfrak{F}} \times^{\mathsf{RLD}} \mathcal{B}$ is an upper adjoint of $\lambda f \in \mathsf{RLD}(A; B): \operatorname{im} f$ (for every sets A, B).

Proof. We need to prove $\operatorname{im} f \sqsubseteq \mathcal{B} \Leftrightarrow f \sqsubseteq 1^{\mathfrak{F}} \times^{\mathsf{RLD}} \mathcal{B}$ what is obvious. □

Corollary 7.39. Image and domain of reloids preserve joins.

Proof. By properties of Galois connections and duality. □

7.5 Categories of reloids

I will define two categories, the *category of reloids* and the *category of reloid triples*.

The *category of reloids* is defined as follows:

- Objects are small sets.
- The set of morphisms from a set A to a set B is $\mathsf{RLD}(A; B)$.
- The composition is the composition of reloids.
- Identity morphism for a set is the identity reloid for that set.

To show it is really a category is trivial.

The *category of reloid triples* is defined as follows:

- Objects are filters on small sets.
- The morphisms from a filter $\mathcal{A}$ to a filter $\mathcal{B}$ are triples $(\mathcal{A}; \mathcal{B}; f)$ where $f \in \mathsf{RLD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{B}))$ and $\operatorname{dom} f \sqsubseteq \mathcal{A}$, $\operatorname{im} f \sqsubseteq \mathcal{B}$.
- The composition is defined by the formula $(\mathcal{B}; \mathcal{C}; g) \circ (\mathcal{A}; \mathcal{B}; f) = (\mathcal{A}; \mathcal{C}; g \circ f)$.
- Identity morphism for a filter $\mathcal{A}$ is $\operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}}$.

To prove that it is really a category is trivial.

7.6 Monovalued and injective reloids

Following the idea of definition of monovalued morphism let's call *monovalued* such a reloid f that $f \circ f^{-1} \sqsubseteq \operatorname{id}^{\mathsf{RLD}}_{\operatorname{im} f}$.

Similarly, I will call a reloid *injective* when $f^{-1} \circ f \sqsubseteq \operatorname{id}^{\mathsf{RLD}}_{\operatorname{dom} f}$.

Obvious 7.40. A reloid f is

- monovalued iff $f \circ f^{-1} \sqsubseteq \operatorname{id}^{\mathsf{RLD}(\operatorname{Dst} f)}$;
- injective iff $f^{-1} \circ f \sqsubseteq \operatorname{id}^{\mathsf{RLD}(\operatorname{Src} f)}$.

In other words, a reloid is monovalued (injective) when it is a monovalued (injective) morphism of the category of reloids.

Monovaluedness is dual of injectivity.

Obvious 7.41.

1. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of reloid triples is monovalued iff the reloid f is monovalued.
2. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of reloid triples is injective iff the reloid f is injective.

Theorem 7.42.

1. A reloid f is a monovalued iff there exists a function (monovalued binary relation) $F \in \operatorname{GR} f$.
2. A reloid f is a injective iff there exists an injective binary relation $F \in \operatorname{GR} f$.