Chapter 7
Reloids

7.1 Basic definitions

Definition 7.1. I call a *reloid* from a set A to a set B a triple $(A; B; F)$ where $F \in \mathfrak{F}(A \times B)$.

Definition 7.2. *Source* and *destination* of every reloid $(A; B; F)$ are defined as

$$\operatorname{Src}(A; B; F) = A \quad \text{and} \quad \operatorname{Dst}(A; B; F) = B.$$

I will denote $\mathsf{RLD}(A; B)$ the set of reloids from A to B.
I will denote RLD the set of all reloids (for small sets).

Definition 7.3. $\operatorname{GR}(A; B; F) \stackrel{\text{def}}{=} F$, $\operatorname{xyGR}(A; B; F) \stackrel{\text{def}}{=} \{(A; B; K) \mid K \in F\}$ for every reloid $(A; B; F)$. Note that $\operatorname{xyGR}(A; B; F)$ is a set of morphisms of the category Rel.

Definition 7.4.

- $\uparrow^{\mathsf{RLD}(A;B)} f \stackrel{\text{def}}{=} (A; B; \uparrow^{A \times B} f)$ for every relation $f \in \mathscr{P}(A \times B)$.
- $\uparrow^{\mathsf{RLD}} f = (\operatorname{Src} f; \operatorname{Dst} f; \uparrow^{\operatorname{Src} f \times \operatorname{Dst} f} \operatorname{GR} f)$ for every Rel-morphism f.

Definition 7.5. I call members of a set $\langle \uparrow^{\mathsf{RLD}} \rangle \operatorname{Rel}(A; B)$ as *principal* reloids.

Reloids are a generalization of uniform spaces. Also reloids are generalization of binary relations.

Definition 7.6. The *reverse* reloid of a reloid is defined by the formula

$$(A; B; F)^{-1} = (B; A; \{K^{-1} \mid K \in F\}).$$

Note 7.7. The reverse reloid is *not* an inverse in the sense of group theory or category theory.

Reverse reloid is a generalization of conjugate quasi-uniformity.

Definition 7.8. Every set $\mathsf{RLD}(A; B)$ is a poset by the formula $f \sqsubseteq g \Leftrightarrow \operatorname{GR} f \sqsubseteq \operatorname{GR} g$. We will apply lattice operations to subsets of $\mathsf{RLD}(A; B)$ without explicitly mentioning $\mathsf{RLD}(A; B)$.

Obvious 7.9. The poset $\mathsf{RLD}(A; B)$ is isomorphic to the poset $\mathfrak{F}(A \times B)$ for every sets A, B.

7.2 Composition of reloids

Definition 7.10. Reloids f and g are *composable* when $\operatorname{Dst} f = \operatorname{Src} g$.

Definition 7.11. *Composition* of (composable) reloids is defined by the formula

$$g \circ f = \bigsqcap \{\uparrow^{\mathsf{RLD}} (G \circ F) \mid F \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g\}.$$

Obvious 7.12. Composition of reloids is a reloid.