In other words, a funcoid is monovalued (injective) when it is a monovalued (injective) morphism of the category of funcoids. Monovaluedness is dual of injectivity.

Obvious 6.146.

1. A morphism $(\mathcal{A};\mathcal{B};f)$ of the category of funcoid triples is monovalued iff the funcoid f is monovalued.
2. A morphism $(\mathcal{A};\ \mathcal{B};\ f)$ of the category of funcoid triples is injective iff the funcoid f is injective.

Theorem 6.147. The following statements are equivalent for a funcoid f:

1. f is monovalued.
2. $\forall a\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}: \langle f\rangle a\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}\cup\{0^{\mathfrak{F}(\operatorname{Dst} f)}\}$.
3. $\forall\mathcal{I},\mathcal{J}\in\mathfrak{F}(\operatorname{Dst} f): \langle f^{-1}\rangle(\mathcal{I}\sqcap\mathcal{J})=\langle f^{-1}\rangle\mathcal{I}\sqcap\langle f^{-1}\rangle\mathcal{J}$.
4. $\forall I,J\in\mathscr{P}(\operatorname{Dst} f): \langle f^{-1}\rangle^*(I\cap J)=\langle f^{-1}\rangle^*I\sqcap\langle f^{-1}\rangle^*J$.

Proof.

(2)⇒(3). Let $a\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}$, $\langle f\rangle a=b$. Then because $b\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}\cup\{0^{\mathfrak{F}(\operatorname{Dst} f)}\}$

$$(\mathcal{I}\sqcap\mathcal{J})\sqcap b\neq 0^{\mathfrak{F}(\operatorname{Dst} f)}\Leftrightarrow\mathcal{I}\sqcap b\neq 0^{\mathfrak{F}(\operatorname{Dst} f)}\wedge\mathcal{J}\sqcap b\neq 0^{\mathfrak{F}(\operatorname{Dst} f)};$$
$$a\,[f]\,\mathcal{I}\sqcap\mathcal{J}\Leftrightarrow a\,[f]\,\mathcal{I}\wedge a\,[f]\,\mathcal{J};$$
$$\mathcal{I}\sqcap\mathcal{J}\,[f^{-1}]\,a\Leftrightarrow\mathcal{I}\,[f^{-1}]\,a\wedge\mathcal{J}\,[f^{-1}]\,a;$$
$$a\sqcap\langle f^{-1}\rangle(\mathcal{I}\sqcap\mathcal{J})\neq 0^{\mathfrak{F}(\operatorname{Src} f)}\Leftrightarrow a\sqcap\langle f^{-1}\rangle\mathcal{I}\neq 0^{\mathfrak{F}(\operatorname{Src} f)}\wedge a\sqcap\langle f^{-1}\rangle\mathcal{J}\neq 0^{\mathfrak{F}(\operatorname{Src} f)};$$
$$\langle f^{-1}\rangle(\mathcal{I}\sqcap\mathcal{J})=\langle f^{-1}\rangle\mathcal{I}\sqcap\langle f^{-1}\rangle\mathcal{J}.$$

(3)⇒(1). $\langle f^{-1}\rangle a\sqcap\langle f^{-1}\rangle b=\langle f^{-1}\rangle(a\sqcap b)=\langle f^{-1}\rangle 0^{\mathfrak{F}(\operatorname{Dst} f)}=0^{\mathfrak{F}(\operatorname{Src} f)}$ for every two distinct ultrafilters a and b on $\operatorname{Dst} f$. This is equivalent to $\neg(\langle f^{-1}\rangle a\ [f]\ b)$; $b\asymp\langle f\rangle\langle f^{-1}\rangle a$; $b\asymp\langle f\circ f^{-1}\rangle a$; $\neg(a\,[f\circ f^{-1}]\,b)$. So $a\,[f\circ f^{-1}]\,b\Rightarrow a=b$ for every ultrafilters a and b. This is possible only when $f\circ f^{-1}\sqsubseteq\operatorname{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$.

(4)⇒(3). $\langle f^{-1}\rangle(\mathcal{I}\sqcap\mathcal{J})=\bigsqcap\langle\langle f^{-1}\rangle^*\rangle(\mathcal{I}\sqcap\mathcal{J})=\bigsqcap\langle\langle f^{-1}\rangle^*\rangle\{I\cap J\mid I\in\mathcal{I},\ J\in\mathcal{J}\}=\bigsqcap\{\langle f^{-1}\rangle^*(I\cap J)\mid I\in\mathcal{I},J\in\mathcal{J}\}=\bigsqcap\{\langle f^{-1}\rangle^*I\sqcap\langle f^{-1}\rangle^*J\mid I\in\mathcal{I},J\in\mathcal{J}\}=\bigsqcap\{\langle f^{-1}\rangle^*I\mid I\in\mathcal{I}\}\sqcap\bigsqcap\{\langle f^{-1}\rangle^*J\mid J\in\mathcal{J}\}=\langle f^{-1}\rangle\mathcal{I}\sqcap\langle f^{-1}\rangle\mathcal{J}$.

(3)⇒(4). Obvious.

¬(2)⇒¬(1). Suppose $\langle f\rangle a\notin\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}\cup\{0^{\mathfrak{F}(\operatorname{Dst} f)}\}$ for some $a\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}$. Then there exist two atomic filters p and q on $\operatorname{Dst} f$ such that $p\neq q$ and $\langle f\rangle a\sqsupseteq p\wedge\langle f\rangle a\sqsupseteq q$. Consequently $p\not\asymp\langle f\rangle a$; $a\not\asymp\langle f^{-1}\rangle p$; $a\sqsubseteq\langle f^{-1}\rangle p$; $\langle f\circ f^{-1}\rangle p=\langle f\rangle\langle f^{-1}\rangle p\sqsupseteq\langle f\rangle a\sqsupseteq q$; $\langle f\circ f^{-1}\rangle p\not\sqsubseteq p$ and $\langle f\circ f^{-1}\rangle p\neq 0^{\mathfrak{F}(\operatorname{Dst} f)}$. So it cannot be $f\circ f^{-1}\sqsubseteq\operatorname{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$. □

Corollary 6.148. A binary relation corresponds to a monovalued funcoid iff it is a function.

Proof. Because $\forall I,\ J\in\mathscr{P}(\operatorname{im}\ f): \langle f^{-1}\rangle^*(I\cap J)=\langle f^{-1}\rangle^*I\sqcap\langle f^{-1}\rangle^*J$ is true for a funcoid f corresponding to a binary relation if and only if it is a function. □

Remark 6.149. This corollary can be reformulated as follows: For binary relations (principal funcoids) the classic concept of monovaluedness and monovaluedness in the above defined sense of monovaluedness of a funcoid are the same.

Proposition 6.150. Every monovalued funcoid is metamonovalued.

Proof. $\langle(\bigsqcap G)\circ f\rangle x=\langle\bigsqcap G\rangle\langle f\rangle x=\bigsqcap_{g\in G}\langle g\rangle\langle f\rangle x=\bigsqcap_{g\in G}\langle g\circ f\rangle x=\langle\bigsqcap_{g\in G}(g\circ f)\rangle x$ for every ultrafilter $x\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}$. Thus $(\bigsqcap G)\circ f=\bigsqcap_{g\in G}(g\circ f)$. □

Corollary 6.151. Every injective funcoid is metainjective.