Proposition 6.139. For every composable funcoids f and g

1. $\operatorname{Compl}(g\circ(\operatorname{Compl} f))=\operatorname{Compl}(g\circ f)$;
2. $\operatorname{CoCompl}((\operatorname{CoCompl} g)\circ f)=\operatorname{CoCompl}(g\circ f)$.

Proof.

1. $\langle g\circ(\operatorname{Compl} f)\rangle^*\{x\}=\langle g\rangle\langle\operatorname{Compl} f\rangle^*\{x\}=\langle g\rangle\langle f\rangle^*\{x\}=\langle g\circ f\rangle^*\{x\}$.
   Thus $\operatorname{Compl}(g\circ(\operatorname{Compl} f))=\operatorname{Compl}(g\circ f)$.
2. $(\operatorname{Compl}(g\circ(\operatorname{Compl} f)))^{-1}=(\operatorname{Compl}(g\circ f))^{-1}$; $\operatorname{CoCompl}(g\circ(\operatorname{Compl} f))^{-1}=\operatorname{CoCompl}(g\circ f)^{-1}$; $\operatorname{CoCompl}((\operatorname{Compl} f)^{-1}\circ g^{-1})=\operatorname{CoCompl}(f^{-1}\circ g^{-1})$; $\operatorname{CoCompl}((\operatorname{CoCompl} f^{-1})\circ g^{-1})=\operatorname{CoCompl}(f^{-1}\circ g^{-1})$. After variable replacement $\operatorname{CoCompl}((\operatorname{CoCompl} g)\circ f)=\operatorname{CoCompl}(g\circ f)$. □

6.13.1.1 Open maps

Definition 6.140. An *open map* from a topological space to a topological space is a function which maps open sets into open sets.

An obvious generalization of this is *open map* f from an endofuncoid μ to an endofuncoid ν, which is by definition a function (or rather a principal, entirely defined, monovalued funcoid) from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ such that

$$\forall x\in\operatorname{Ob}\mu, V\in\langle\mu\rangle^*\{x\}: \langle f\rangle^* V\sqsupseteq\langle\nu\rangle\langle f\rangle^*\{x\}.$$

This formula is equivalent (exercise!) to

$$\forall x\in\operatorname{Ob}\mu: \langle f\rangle\langle\mu\rangle^*\{x\}\sqsupseteq\langle\nu\rangle\langle f\rangle^*\{x\}.$$

It can be abstracted/simplified further (now for an *arbitrary* funcoid f from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$):

$$\operatorname{Compl}(f\circ\mu)\sqsupseteq\operatorname{Compl}(\nu\circ f).$$

Definition 6.141. An *open funcoid* from an endofuncoid μ to an endofuncoid ν is a funcoid f from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ such that $\operatorname{Compl}(f\circ\mu)\sqsupseteq\operatorname{Compl}(\nu\circ f)$.

Theorem 6.142. Let μ, ν, π be endofuncoids. Let f be a co-complete open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ and g is an open funcoid from $\operatorname{Ob}\nu$ to $\operatorname{Ob}\pi$. Then $g\circ f$ is an open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\pi$.

Proof. Let $\operatorname{Compl}(f\circ\mu)\sqsupseteq\operatorname{Compl}(\nu\circ f)$ and $\operatorname{Compl}(g\circ\nu)\sqsupseteq\operatorname{Compl}(\pi\circ g)$.

$\operatorname{Compl}(g\circ f\circ\mu)\sqsupseteq\operatorname{Compl}(g\circ\operatorname{Compl}(f\circ\mu))\sqsupseteq\operatorname{Compl}(g\circ\operatorname{Compl}(\nu\circ f))=\operatorname{Compl}(g\circ\operatorname{Compl}(\nu)\circ f)=\operatorname{Compl}(g\circ\operatorname{Compl}(\nu))\circ f=\operatorname{Compl}(g\circ\nu)\circ f\sqsupseteq\operatorname{Compl}(\pi\circ g)\circ f=\operatorname{Compl}(\pi\circ g\circ f)$. □

Obvious 6.143. A funcoid f is open iff $f\circ\mu\sqsupseteq\operatorname{Compl}(\nu\circ f)$.

Corollary 6.144. A co-complete funcoid f is open iff $f\circ\mu\sqsupseteq(\operatorname{Compl}\nu)\circ f$. Thus f is open iff it is a continuous morphism from μ to $\operatorname{Compl}\nu$ *with the reverse order of funcoids*. (See a definition of a continuous morphism below.)

6.14 Monovalued and injective funcoids

Following the idea of definition of monovalued morphism let's call *monovalued* such a funcoid f that $f\circ f^{-1}\sqsubseteq\operatorname{id}^{\mathsf{FCD}}_{\operatorname{im} f}$.

Similarly, I will call a funcoid *injective* when $f^{-1}\circ f\sqsubseteq\operatorname{id}^{\mathsf{FCD}}_{\operatorname{dom} f}$.

Obvious 6.145. A funcoid f is:

- monovalued iff $f\circ f^{-1}\sqsubseteq\operatorname{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$;
- injective iff $f^{-1}\circ f\sqsubseteq\operatorname{id}^{\mathsf{FCD}(\operatorname{Src} f)}$.