and we have (6.10) obviously.

⇐. Let (6.10) hold. Then $G(\alpha) = \bigsqcup \operatorname{atoms} G(\alpha)$ and thus

$$f = \bigsqcup \{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{FCD}} b \mid \alpha \in \operatorname{Src} f, b \in \operatorname{atoms} G(\alpha)\}$$

and so f is complete. □

Theorem 6.115.

1. For a complete funcoid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$ such that

$$f = \bigsqcup \{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{FCD}} F(\alpha) \mid \alpha \in \operatorname{Src} f\}.$$

2. For a co-complete funcoid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Src} f)^{\operatorname{Dst} f}$ such that

$$f = \bigsqcup \{F(\alpha) \times^{\mathsf{FCD}} \uparrow^{\operatorname{Dst} f}\{\alpha\} \mid \alpha \in \operatorname{Dst} f\}.$$

Proof. We will prove only the first as the second is similar. Let

$$f = \bigsqcup \{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{FCD}} F(\alpha) \mid \alpha \in \operatorname{Src} f\} = \bigsqcup \{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{FCD}} G(\alpha) \mid \alpha \in \operatorname{Src} f\}$$

for some $F, G \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$. We need to prove $F = G$. Let $\beta \in \operatorname{Src} f$.

$$\langle f\rangle^*\{\beta\} = \bigsqcup \{\langle\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{FCD}} F(\alpha)\rangle^*\{\beta\} \mid \alpha \in \operatorname{Src} f\} = F(\beta).$$

Similarly $\langle f\rangle^*\{\beta\} = G(\beta)$. So $F(\beta) = G(\beta)$. □

6.12 Funcoids corresponding to pretopologies

Let Δ be a pretopology on a set U and cl the preclosure corresponding to it (see theorem 5.12).

Both induce a funcoid, I will show that these two funcoids are reverse of each other:

Theorem 6.116. Let f be a complete funcoid defined by the formula $\langle f\rangle^*\{x\} = \Delta(x)$ for every $x \in U$, let g be a co-complete funcoid defined by the formula $\langle g\rangle^* X = \uparrow^U \operatorname{cl}(X)$ for every $X \in \mathscr{P}U$. Then $g = f^{-1}$.

Remark 6.117. It is obvious that funcoids f and g exist.

Proof. $X\, [g]^*\, Y \Leftrightarrow \uparrow^U Y \not\asymp \langle g\rangle \uparrow^U X \Leftrightarrow Y \not\asymp \operatorname{cl}(X) \Leftrightarrow \exists y \in Y : \Delta(y) \not\asymp \uparrow^U X \Leftrightarrow \exists y \in Y : \langle f\rangle^*\{y\} \not\asymp \uparrow^U X \Leftrightarrow$ (proposition 4.194 and properties of complete funcoids)$\Leftrightarrow \langle f\rangle^* Y \not\asymp \uparrow^U X \Leftrightarrow Y\, [f]^*\, X$.

So $g = f^{-1}$. □

6.13 Completion of funcoids

Theorem 6.118. $\operatorname{Cor} f = \operatorname{Cor}' f$ for an element f of a filtrator of funcoids.

Proof. By theorems 4.34 and 6.108. □

Definition 6.119. *Completion* of a funcoid $f \in \mathsf{FCD}(A; B)$ is the complete funcoid $\operatorname{Compl} f \in \mathsf{FCD}(A; B)$ defined by the formula $\langle \operatorname{Compl} f\rangle^*\{\alpha\} = \langle f\rangle^*\{\alpha\}$ for $\alpha \in \operatorname{Src} f$.

Definition 6.120. *Co-completion* of a funcoid f is defined by the formula

$$\operatorname{CoCompl} f = (\operatorname{Compl} f^{-1})^{-1}.$$

Obvious 6.121. $\operatorname{Compl} f \sqsubseteq f$ and $\operatorname{CoCompl} f \sqsubseteq f$.