Proof. By theorem 3.21. □

Remark 6.87. For more ways to characterize (atomic) separability of the lattice of funcoids see subsections “Separation subsets and full stars” and “Atomically separable lattices”.

Corollary 6.88. The lattice $\mathsf{FCD}(A;B)$ is an atomistic lattice.

Proof. Let $f\in\mathsf{FCD}(A;B)$. Suppose contrary to the statement to be proved that $\bigsqcup \operatorname{atoms} f\sqsubset f$. Then there exists $a\in\operatorname{atoms} f$ such that $a\sqcap\bigsqcup \operatorname{atoms} f=0^{\mathsf{FCD}(A;B)}$ what is impossible. □

Proposition 6.89. $\operatorname{atoms}(f\sqcup g)=\operatorname{atoms} f\cup\operatorname{atoms} g$ for every funcoids f, $g\in\mathsf{FCD}(A;B)$ (for every sets A, B).

Proof. $a\times^{\mathsf{FCD}}b\not\asymp f\sqcup g\Leftrightarrow a\,[f\sqcup g]\,b\Leftrightarrow a\,[f]\,b\vee a\,[g]\,b\Leftrightarrow a\times^{\mathsf{FCD}}b\not\asymp f\vee a\times^{\mathsf{FCD}}b\not\asymp g$ for every atomic filters a and b. □

Theorem 6.90. For every $f,g,h\in\mathsf{FCD}(A;B)$, $R\in\mathscr{P}\mathsf{FCD}(A;B)$ (for every sets A and B)

1. $f\sqcap(g\sqcup h)=(f\sqcap g)\sqcup(f\sqcap h)$;
2. $f\sqcup\bigsqcap R=\bigsqcap\langle f\sqcup\rangle R$.

Proof. We will take into account that the lattice of funcoids is an atomistic lattice.

1. $\operatorname{atoms}(f\sqcap(g\sqcup h))=\operatorname{atoms} f\cap\operatorname{atoms}(g\sqcup h)=\operatorname{atoms} f\cap(\operatorname{atoms} g\cup\operatorname{atoms} h)=(\operatorname{atoms} f\cap\operatorname{atoms} g)\cup(\operatorname{atoms} f\cap\operatorname{atoms} h)=\operatorname{atoms}(f\sqcap g)\cup\operatorname{atoms}(f\sqcap h)=\operatorname{atoms}((f\sqcap g)\sqcup(f\sqcap h))$.

2. $\operatorname{atoms}(f\sqcup\bigsqcap R)=\operatorname{atoms} f\cup\operatorname{atoms}\bigsqcap R=\operatorname{atoms} f\cup\bigcap\langle\operatorname{atoms}\rangle R=\bigcap\langle(\operatorname{atoms} f)\cup\rangle\langle\operatorname{atoms}\rangle R=\bigcap\langle\operatorname{atoms}\rangle\langle f\sqcup\rangle R=\operatorname{atoms}\bigsqcap\langle f\sqcup\rangle R$. (Used the following equality.)

$$\begin{aligned}\langle(\operatorname{atoms} f)\cup\rangle\langle\operatorname{atoms}\rangle R &=\\ \{(\operatorname{atoms} f)\cup A\mid A\in\langle\operatorname{atoms}\rangle R\} &=\\ \{(\operatorname{atoms} f)\cup A\mid\exists C\in R: A=\operatorname{atoms} C\} &=\\ \{(\operatorname{atoms} f)\cup(\operatorname{atoms} C)\mid C\in R\} &=\\ \{\operatorname{atoms}(f\sqcup C)\mid C\in R\} &=\\ \{\operatorname{atoms} B\mid\exists C\in R: B=f\sqcup C\} &=\\ \{\operatorname{atoms} B\mid B\in\langle f\sqcup\rangle R\} &=\\ \langle\operatorname{atoms}\rangle\langle f\sqcup\rangle R.&\end{aligned}$$

□

Note that distributivity of the lattice of funcoids is proved through using atoms of this lattice. I have never seen such method of proving distributivity.

Corollary 6.91. The lattice $\mathsf{FCD}(A;B)$ is co-brouwerian (for every sets A, B).

Conjecture 6.92. Distributivity of the lattice $\mathsf{FCD}(A;B)$ of funcoids (for arbitrary sets A and B) is not provable in ZF (without axiom of choice).

The next proposition is one more (among the theorem 6.44) generalization for funcoids of composition of relations.

Proposition 6.93. For every composable funcoids f, g

$$\operatorname{atoms}(g\circ f)=\left\{x\times^{\mathsf{FCD}}z\mid\begin{array}{l}x\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, z\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} g)},\\ \exists y\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}:(x\times^{\mathsf{FCD}}y\in\operatorname{atoms} f\wedge y\times^{\mathsf{FCD}}z\in\operatorname{atoms} g)\end{array}\right\}.$$

Proof. $x\times^{\mathsf{FCD}}z\not\asymp g\circ f\Leftrightarrow x\,[g\circ f]\,z\Leftrightarrow\exists y\in\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}:(x\times^{\mathsf{FCD}}y\not\asymp f\wedge y\times^{\mathsf{FCD}}z\not\asymp g)$ (it was used the theorem 6.44). □

Corollary 6.94. $g\circ f=\bigsqcup\{G\circ F\mid F\in\operatorname{atoms} f, G\in\operatorname{atoms} g\}$ for every composable funcoids f, g.