Proof. $f \sqcap \left(\mathcal{A} \times^{\mathsf{FCD}} 1^{\mathfrak{F}(\mathrm{Dst}\, f)}\right) = \mathrm{id}^{\mathsf{FCD}}_{1^{\mathfrak{F}(\mathrm{Dst}\, f)}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} = f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} = f|_{\mathcal{A}}$. □

Corollary 6.77. $f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow \mathcal{A} [f] \mathcal{B}$ for every funcoid f, $\mathcal{A} \in \mathfrak{F}(\mathrm{Src}\, f)$, $\mathcal{B} \in (\mathrm{Dst}\, f)$.

Proof. $f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow \langle f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})\rangle^{*}(\mathrm{Src}\, f) \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\rangle^{*}(\mathrm{Src}\, f) \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}}\rangle \langle f\rangle \langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\rangle 1^{\mathfrak{F}(\mathrm{Src}\, f)} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \mathcal{B} \sqcap \langle f\rangle \left(\mathcal{A} \sqcap 1^{\mathfrak{F}(\mathrm{Src}\, f)}\right) \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \mathcal{B} \sqcap \langle f\rangle \mathcal{A} \neq 0^{\mathfrak{F}(\mathrm{Dst}\, f)} \Leftrightarrow \mathcal{A} [f] \mathcal{B}$. □

Corollary 6.78. Every filtrator of funcoids is star-separable.

Proof. The set of funcoidal products of principal filters is a separation subset of the lattice of funcoids. □

Theorem 6.79. Let A, B be sets. If $S \in \mathscr{P}(\mathfrak{F}(A) \times \mathfrak{F}(B))$ then

$$\bigsqcap \{\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \mid (\mathcal{A}; \mathcal{B}) \in S\} = \bigsqcap \mathrm{dom}\, S \times^{\mathsf{FCD}} \bigsqcap \mathrm{im}\, S.$$

Proof. If $x \in \mathrm{atoms}^{\mathfrak{F}(A)}$ then by theorem 6.68

$$\left\langle \bigsqcap \{\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \mid (\mathcal{A}; \mathcal{B}) \in S\}\right\rangle x = \bigsqcap \{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x \mid (\mathcal{A}; \mathcal{B}) \in S\}.$$

If $x \not\asymp \bigsqcap \mathrm{dom}\, S$ then

$$\forall (\mathcal{A}; \mathcal{B}) \in S: \left(x \sqcap \mathcal{A} \neq 0^{\mathfrak{F}(A)} \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x = \mathcal{B}\right);$$
$$\{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x \mid (\mathcal{A}; \mathcal{B}) \in S\} = \mathrm{im}\, S;$$

if $x \asymp \bigsqcap \mathrm{dom}\, S$ then

$$\exists (\mathcal{A}; \mathcal{B}) \in S: \left(x \sqcap \mathcal{A} = 0^{\mathfrak{F}(A)} \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x = 0^{\mathfrak{F}(B)}\right);$$
$$\{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x \mid (\mathcal{A}; \mathcal{B}) \in S\} \ni 0^{\mathfrak{F}(B)}.$$

So

$$\left\langle \bigsqcap \{\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \mid (\mathcal{A}; \mathcal{B}) \in S\}\right\rangle x = \begin{cases} \bigsqcap \mathrm{im}\, S & \text{if } x \not\asymp \bigsqcap \mathrm{dom}\, S \\ 0^{\mathfrak{F}(B)} & \text{if } x \asymp \bigsqcap \mathrm{dom}\, S. \end{cases}$$

From this the statement of the theorem follows. □

Corollary 6.80. For every $\mathcal{A}_0, \mathcal{A}_1 \in \mathfrak{F}(A)$, $\mathcal{B}_0, \mathcal{B}_1 \in \mathfrak{F}(B)$ (for every sets A, B)

$$(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = (\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1).$$

Proof. $(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = \bigsqcap \{\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0, \mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1\}$ what is by the last theorem equal to $(\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1)$. □

Theorem 6.81. If A, B are sets and $\mathcal{A} \in \mathfrak{F}(A)$ then $\mathcal{A} \times^{\mathsf{FCD}}$ is a complete homomorphism from the lattice $\mathfrak{F}(B)$ to the lattice $\mathsf{FCD}(A; B)$, if also $\mathcal{A} \neq 0^{\mathfrak{F}(A)}$ then it is an order embedding.

Proof. Let $S \in \mathscr{P}\mathfrak{F}(B)$, $X \in \mathscr{P}A$, $x \in \mathrm{atoms}^{\mathfrak{F}(A)}$.

$$\begin{aligned}
\left\langle \bigsqcup \langle \mathcal{A} \times^{\mathsf{FCD}}\rangle S\right\rangle^{*} X &= \bigsqcup \{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle^{*} X \mid \mathcal{B} \in S\} \\
&= \begin{cases} \bigsqcup S & \text{if } X \in \partial \mathcal{A} \\ 0^{\mathfrak{F}(B)} & \text{if } X \notin \partial \mathcal{A} \end{cases} \\
&= \left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigsqcup S\right\rangle^{*} X; \\
\left\langle \bigsqcap \langle \mathcal{A} \times^{\mathsf{FCD}}\rangle S\right\rangle x &= \bigsqcap \{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}\rangle x \mid \mathcal{B} \in S\} \\
&= \begin{cases} \bigsqcap S & \text{if } x \not\asymp \mathcal{A} \\ 0^{\mathfrak{F}(B)} & \text{if } x \asymp \mathcal{A} \end{cases} \\
&= \left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigsqcap S\right\rangle x.
\end{aligned}$$