The reverse is obvious. □

Corollary 6.66. Let f be a funcoid.

- The value of f can be restored knowing $\langle f\rangle|_{\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}}$.
- The value of f can be restored knowing $[f]|_{\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}\times\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}}$.

Theorem 6.67. Let A and B be sets.

1. A function $\alpha\in\mathfrak{F}(B)^{\operatorname{atoms}^{\mathfrak{F}(A)}}$ such that (for every $a\in\operatorname{atoms}^{\mathfrak{F}(A)}$)

$$\alpha a\sqsubseteq\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle\circ\operatorname{atoms}\circ\uparrow^A\right\rangle a \tag{6.6}$$

can be continued to the function $\langle f\rangle$ for a unique $f\in\mathsf{FCD}(A;B)$;

$$\langle f\rangle\mathcal{X}=\bigsqcup\langle\alpha\rangle\operatorname{atoms}\mathcal{X} \tag{6.7}$$

for every $\mathcal{X}\in\mathfrak{F}(A)$.

2. A relation $\delta\in\mathscr{P}\left(\operatorname{atoms}^{\mathfrak{F}(A)}\times\operatorname{atoms}^{\mathfrak{F}(B)}\right)$ such that (for every $a\in\operatorname{atoms}^{\mathfrak{F}(A)}$, $b\in\operatorname{atoms}^{\mathfrak{F}(B)}$)

$$\forall X\in a,Y\in b\exists x\in\operatorname{atoms}\uparrow^A X,y\in\operatorname{atoms}\uparrow^B Y: x\,\delta\,y\Rightarrow a\,\delta\,b \tag{6.8}$$

can be continued to the relation $[f]$ for a unique $f\in\mathsf{FCD}(A;B)$;

$$\mathcal{X}\,[f]\,\mathcal{Y}\Leftrightarrow\exists x\in\operatorname{atoms}\mathcal{X},y\in\operatorname{atoms}\mathcal{Y}: x\,\delta\,y \tag{6.9}$$

for every $\mathcal{X}\in\mathfrak{F}(A)$, $\mathcal{Y}\in\mathfrak{F}(B)$.

Proof. Existence of no more than one such funcoids and formulas (6.7) and (6.9) follow from the previous theorem.

1. Consider the function $\alpha'\in\mathfrak{F}(B)^{\mathscr{P}A}$ defined by the formula (for every $X\in\mathscr{P}A$)

$$\alpha' X=\bigsqcup\langle\alpha\rangle\operatorname{atoms}\uparrow^A X.$$

Obviously $\alpha'\emptyset=0^{\mathfrak{F}(B)}$. For every $I,J\in\mathscr{P}A$

$$\begin{aligned}
\alpha'(I\cup J) &= \bigsqcup\langle\alpha\rangle\operatorname{atoms}\uparrow^A(I\cup J)\\
&= \bigsqcup\langle\alpha\rangle(\operatorname{atoms}\uparrow^A I\cup\operatorname{atoms}\uparrow^A J)\\
&= \bigsqcup(\langle\alpha\rangle\operatorname{atoms}\uparrow^A I\cup\langle\alpha\rangle\operatorname{atoms}\uparrow^A J)\\
&= \bigsqcup\langle\alpha\rangle\operatorname{atoms}\uparrow^A I\sqcup\bigsqcup\langle\alpha\rangle\operatorname{atoms}\uparrow^A J\\
&= \alpha' I\sqcup\alpha' J.
\end{aligned}$$

Let continue α' till a funcoid f (by the theorem 6.28): $\langle f\rangle\mathcal{X}=\bigsqcap\langle\alpha'\rangle\mathcal{X}$.

Let's prove the reverse of (6.6):

$$\begin{aligned}
\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle\circ\operatorname{atoms}\circ\uparrow^A\right\rangle a &= \bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle\right\rangle\langle\operatorname{atoms}\rangle\langle\uparrow^A\rangle a\\
&\sqsubseteq \bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle\right\rangle\{\{a\}\}\\
&= \bigsqcap\left\{\left(\bigsqcup\circ\langle\alpha\rangle\right)\{a\}\right\}\\
&= \bigsqcap\left\{\bigsqcup\langle\alpha\rangle\{a\}\right\}\\
&= \bigsqcap\left\{\bigsqcup\{\alpha a\}\right\}=\bigsqcap\{\alpha a\}=\alpha a.
\end{aligned}$$

Finally,

$$\alpha a=\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle\circ\operatorname{atoms}\circ\uparrow^A\right\rangle a=\bigsqcap\langle\alpha'\rangle a=\langle f\rangle a,$$

so $\langle f\rangle$ is a continuation of α.

2. Consider the relation $\delta'\in\mathscr{P}(\mathscr{P}A\times\mathscr{P}B)$ defined by the formula (for every $X\in\mathscr{P}A$, $Y\in\mathscr{P}B$)

$$X\,\delta'\,Y\Leftrightarrow\exists x\in\operatorname{atoms}\uparrow^A X,y\in\operatorname{atoms}\uparrow^B Y: x\,\delta\,y.$$

Obviously $\neg(X\,\delta'\,\emptyset)$ and $\neg(\emptyset\,\delta'\,Y)$.