Proposition 6.50. The restricted identity funcoid is a funcoid.

Proof. We need to prove that $(\mathcal{A}\sqcap\mathcal{X})\sqcap\mathcal{Y}\neq 0^{\mathfrak{F}(A)}\Leftrightarrow(\mathcal{A}\sqcap\mathcal{Y})\sqcap\mathcal{X}\neq 0^{\mathfrak{F}(A)}$ what is obvious. □

Obvious 6.51.

1. $\left(\mathrm{id}^{\mathsf{FCD}(A)}\right)^{-1}=\mathrm{id}^{\mathsf{FCD}(A)}$;
2. $(\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}})^{-1}=\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}$.

Obvious 6.52. For every $\mathcal{X},\mathcal{Y}\in\mathfrak{F}(A)$

1. $\mathcal{X}\left[\mathrm{id}^{\mathsf{FCD}(A)}\right]\mathcal{Y}\Leftrightarrow\mathcal{X}\sqcap\mathcal{Y}\neq 0^{\mathfrak{F}(A)}$;
2. $\mathcal{X}\left[\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\right]\mathcal{Y}\Leftrightarrow\mathcal{A}\sqcap\mathcal{X}\sqcap\mathcal{Y}\neq 0^{\mathfrak{F}(A)}$.

Definition 6.53. I will define *restricting* of a funcoid f to a filter $\mathcal{A}\in\mathfrak{F}(\mathrm{Src}\,f)$ by the formula

$$f|_{\mathcal{A}}=f\circ\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}.$$

Definition 6.54. *Image* of a funcoid f will be defined by the formula $\mathrm{im}\,f=\langle f\rangle 1^{\mathfrak{F}(\mathrm{Src}\,f)}$.

Domain of a funcoid f is defined by the formula $\mathrm{dom}\,f=\mathrm{im}\,f^{-1}$.

Obvious 6.55. For every binary relation f between sets A and B

1. $\mathrm{im}\uparrow^{\mathsf{FCD}(A;B)}f=\uparrow^B\mathrm{im}\,f$;
2. $\mathrm{dom}\uparrow^{\mathsf{FCD}(A;B)}f=\uparrow^A\mathrm{dom}\,f$.

Proposition 6.56. $\langle f\rangle\mathcal{X}=\langle f\rangle(\mathcal{X}\sqcap\mathrm{dom}\,f)$ for every funcoid f, $\mathcal{X}\in\mathfrak{F}(\mathrm{Src}\,f)$.

Proof. For every $\mathcal{Y}\in\mathfrak{F}(\mathrm{Dst}\,f)$ we have $\mathcal{Y}\sqcap\langle f\rangle(\mathcal{X}\sqcap\mathrm{dom}\,f)\neq 0^{\mathfrak{F}(\mathrm{Dst}\,f)}\Leftrightarrow\mathcal{X}\sqcap\mathrm{dom}\,f\sqcap\langle f^{-1}\rangle\mathcal{Y}\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow\mathcal{X}\sqcap\mathrm{im}\,f^{-1}\sqcap\langle f^{-1}\rangle\mathcal{Y}\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow\mathcal{X}\sqcap\langle f^{-1}\rangle\mathcal{Y}\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow\mathcal{Y}\sqcap\langle f\rangle\mathcal{X}\neq 0^{\mathfrak{F}(\mathrm{dst}\,f)}$. Thus $\langle f\rangle(\mathcal{X}\sqcap\mathrm{dom}\,f)=\langle f\rangle\mathcal{X}$ because the lattice of filters is separable. □

Proposition 6.57. $\langle f\rangle\mathcal{X}=\mathrm{im}(f|_{\mathcal{X}})$ for every funcoid f, $\mathcal{X}\in\mathfrak{F}(\mathrm{Src}\,f)$.

Proof. $\mathrm{im}(f|_{\mathcal{X}})=\langle f\circ\mathrm{id}^{\mathsf{FCD}}_{\mathcal{X}}\rangle 1^{\mathfrak{F}(\mathrm{Src}\,f)}=\langle f\rangle\langle\mathrm{id}^{\mathsf{FCD}}_{\mathcal{X}}\rangle 1^{\mathfrak{F}(\mathrm{Src}\,f)}=\langle f\rangle\left(\mathcal{X}\sqcap 1^{\mathfrak{F}(\mathrm{Src}\,f)}\right)=\langle f\rangle\mathcal{X}$. □

Proposition 6.58. $\mathcal{X}\sqcap\mathrm{dom}\,f\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow\langle f\rangle\mathcal{X}\neq 0^{\mathfrak{F}(\mathrm{Dst}\,f)}$ for every funcoid f and $\mathcal{X}\in\mathfrak{F}(\mathrm{Src}\,f)$.

Proof. $\mathcal{X}\sqcap\mathrm{dom}\,f\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow\mathcal{X}\sqcap\langle f^{-1}\rangle 1^{\mathfrak{F}(\mathrm{Dst}\,f)}\neq 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow 1^{\mathfrak{F}(\mathrm{Dst}\,f)}\sqcap\langle f\rangle\mathcal{X}\neq 0^{\mathfrak{F}(\mathrm{Dst}\,f)}\Leftrightarrow\langle f\rangle\mathcal{X}\neq 0^{\mathfrak{F}(\mathrm{Dst}\,f)}$. □

Corollary 6.59. $\mathrm{dom}\,f=\bigsqcup\left\{a\in\mathrm{atoms}^{\mathfrak{F}(\mathrm{Src}\,f)}\mid\langle f\rangle a\neq 0^{\mathfrak{F}(\mathrm{Dst}\,f)}\right\}$.

Proof. This follows from the fact that $\mathfrak{F}(\mathrm{Src}\,f)$ is an atomistic lattice. □

Proposition 6.60. $\mathrm{dom}(f|_{\mathcal{A}})=\mathcal{A}\sqcap\mathrm{dom}\,f$ for every funcoid f and $\mathcal{A}\in\mathfrak{F}(\mathrm{Src}\,f)$.

Proof. $\mathrm{dom}(f|_{\mathcal{A}})=\mathrm{im}(\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\circ f^{-1})=\langle\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\rangle\langle f^{-1}\rangle 1^{\mathfrak{F}(\mathrm{Dst}\,f)}=\mathcal{A}\sqcap\langle f^{-1}\rangle 1^{\mathfrak{F}(\mathrm{Dst}\,f)}=\mathcal{A}\sqcap\mathrm{dom}\,f$. □

Theorem 6.61. $\mathrm{im}\,f=\bigsqcap\langle\mathrm{im}\rangle\mathrm{up}\,f$ and $\mathrm{dom}\,f=\bigsqcap\langle\mathrm{dom}\rangle\mathrm{up}\,f$ for every funcoid f.

Proof. $\mathrm{im}\,f=\langle f\rangle 1^{\mathfrak{F}(\mathrm{Src}\,f)}=\bigsqcap\left\{\langle F\rangle 1^{\mathfrak{F}(\mathrm{Src}\,f)}\mid F\in\mathrm{up}\,f\right\}=\bigsqcap\{\mathrm{im}\,F\mid F\in\mathrm{up}\,f\}=\bigsqcap\langle\mathrm{im}\rangle\mathrm{up}\,f$.

The second formula follows from symmetry. □

Proposition 6.62. For every composable funcoids f, g:

1. If $\mathrm{im}\,f\sqsupseteq\mathrm{dom}\,g$ then $\mathrm{im}(g\circ f)=\mathrm{im}\,g$.
2. If $\mathrm{im}\,f\sqsubseteq\mathrm{dom}\,g$ then $\mathrm{dom}(g\circ f)=\mathrm{dom}\,f$.