Provided that $[f]=[g]$ we have $\mathcal{Y}\not\asymp\langle f\rangle\mathcal{X}\Leftrightarrow\mathcal{X}\,[f]\,\mathcal{Y}\Leftrightarrow\mathcal{X}\,[g]\,\mathcal{Y}\Leftrightarrow\mathcal{Y}\not\asymp\langle g\rangle\mathcal{X}$ and consequently $\langle f\rangle\mathcal{X}=\langle g\rangle\mathcal{X}$ for every $\mathcal{X}\in\mathfrak{F}(A)$, $\mathcal{Y}\in\mathfrak{F}(B)$ because a set of filters is separable, thus $\langle f\rangle=\langle g\rangle$. □

Proposition 6.12. $\langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\,f)}=0^{\mathfrak{F}(\mathrm{Dst}\,f)}$ for every funcoid f.

Proof. $\mathcal{Y}\not\asymp\langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\,f)}\Leftrightarrow 0^{\mathfrak{F}(\mathrm{Src}\,f)}\not\asymp\langle f^{-1}\rangle\mathcal{Y}\Leftrightarrow 0\Leftrightarrow\mathcal{Y}\not\asymp 0^{\mathfrak{F}(\mathrm{Dst}\,f)}$. Thus $\langle f\rangle 0^{\mathfrak{F}(\mathrm{Src}\,f)}=0^{\mathfrak{F}(\mathrm{Dst}\,f)}$ by separability of filters. □

Proposition 6.13. $\langle f\rangle(\mathcal{I}\sqcup\mathcal{J})=\langle f\rangle\mathcal{I}\sqcup\langle f\rangle\mathcal{J}$ for every funcoid f and $\mathcal{I},\mathcal{J}\in\mathfrak{F}(\mathrm{Src}\,f)$.

Proof.

$$\begin{aligned}
\star\langle f\rangle(\mathcal{I}\sqcup\mathcal{J}) &=\\
\{\mathcal{Y}\in\mathfrak{F}\mid\mathcal{Y}\not\asymp\langle f\rangle(\mathcal{I}\sqcup\mathcal{J})\} &=\\
\{\mathcal{Y}\in\mathfrak{F}\mid\mathcal{I}\sqcup\mathcal{J}\not\asymp\langle f^{-1}\rangle\mathcal{Y}\} &=\\
\{\mathcal{Y}\in\mathfrak{F}\mid\mathcal{I}\not\asymp\langle f^{-1}\rangle\mathcal{Y}\vee\mathcal{J}\not\asymp\langle f^{-1}\rangle\mathcal{Y}\} &=\\
\{\mathcal{Y}\in\mathfrak{F}\mid\mathcal{Y}\not\asymp\langle f\rangle\mathcal{I}\vee\mathcal{Y}\not\asymp\langle f\rangle\mathcal{J}\} &=\\
\{\mathcal{Y}\in\mathfrak{F}\mid\mathcal{Y}\not\asymp\langle f\rangle\mathcal{I}\sqcup\langle f\rangle\mathcal{J}\} &=\\
\star(\langle f\rangle\mathcal{I}\sqcup\langle f\rangle\mathcal{J}).&
\end{aligned}$$

Thus $\langle f\rangle(\mathcal{I}\sqcup\mathcal{J})=\langle f\rangle\mathcal{I}\sqcup\langle f\rangle\mathcal{J}$ because $\mathfrak{F}(\mathrm{Dst}\,f)$ is separable. □

Proposition 6.14. For every $f\in\mathsf{FCD}(A;B)$ for every sets A and B we have:

1. $\mathcal{K}\,[f]\,\mathcal{I}\sqcup\mathcal{J}\Leftrightarrow\mathcal{K}\,[f]\,\mathcal{I}\vee\mathcal{K}\,[f]\,\mathcal{J}$ for every $\mathcal{I},\mathcal{J}\in\mathfrak{F}(B)$, $\mathcal{K}\in\mathfrak{F}(A)$.
2. $\mathcal{I}\sqcup\mathcal{J}\,[f]\,\mathcal{K}\Leftrightarrow\mathcal{I}\,[f]\,\mathcal{K}\vee\mathcal{J}\,[f]\,\mathcal{K}$ for every $\mathcal{I},\mathcal{J}\in\mathfrak{F}(A)$, $\mathcal{K}\in\mathfrak{F}(B)$.

Proof.

1. $\mathcal{K}\,[f]\,\mathcal{I}\sqcup\mathcal{J}\Leftrightarrow(\mathcal{I}\sqcup\mathcal{J})\sqcap\langle f\rangle\mathcal{K}\neq 0^{\mathfrak{F}(B)}\Leftrightarrow\mathcal{I}\sqcap\langle f\rangle\mathcal{K}\neq 0^{\mathfrak{F}(B)}\vee\mathcal{J}\sqcap\langle f\rangle\mathcal{K}\neq 0^{\mathfrak{F}(B)}\Leftrightarrow\mathcal{K}\,[f]\,\mathcal{I}\vee\mathcal{K}\,[f]\,\mathcal{J}$.
2. Similar. □

6.2.1 Composition of funcoids

Definition 6.15. Funcoids f and g are *composable* when $\mathrm{Dst}\,f=\mathrm{Src}\,g$.

Definition 6.16. *Composition* of composable funcoids is defined by the formula

$$(B;C;\alpha_2;\beta_2)\circ(A;B;\alpha_1;\beta_1)=(A;C;\alpha_2\circ\alpha_1;\beta_1\circ\beta_2).$$

Proposition 6.17. If f, g are composable funcoids then $g\circ f$ is a funcoid.

Proof. Let $f=(A;B;\alpha_1;\beta_1)$, $g=(B;C;\alpha_2;\beta_2)$. For every $\mathcal{X}\in\mathfrak{F}(A)$, $\mathcal{Y}\in\mathfrak{F}(C)$ we have

$$\mathcal{Y}\not\asymp(\alpha_2\circ\alpha_1)\mathcal{X}\Leftrightarrow\mathcal{Y}\not\asymp\alpha_2\,\alpha_1\,\mathcal{X}\Leftrightarrow\alpha_1\,\mathcal{X}\not\asymp\beta_2\,\mathcal{Y}\Leftrightarrow\mathcal{X}\not\asymp\beta_1\,\beta_2\,\mathcal{Y}\Leftrightarrow\mathcal{X}\not\asymp(\beta_1\circ\beta_2)\mathcal{Y}.$$

So $(A;C;\alpha_2\circ\alpha_1;\beta_1\circ\beta_2)$ is a funcoid. □

Obvious 6.18. $\langle g\circ f\rangle=\langle g\rangle\circ\langle f\rangle$ for every composable funcoids f and g.

Proposition 6.19. $(h\circ g)\circ f=h\circ(g\circ f)$ for every composable funcoids f, g, h.

Proof. $\langle(h\circ g)\circ f\rangle=\langle h\circ g\rangle\circ\langle f\rangle=(\langle h\rangle\circ\langle g\rangle)\circ\langle f\rangle=\langle h\rangle\circ(\langle g\rangle\circ\langle f\rangle)=\langle h\rangle\circ\langle g\circ f\rangle=\langle h\circ(g\circ f)\rangle$. □

Theorem 6.20. $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$ for every composable funcoids f and g.

Proof. $\langle(g\circ f)^{-1}\rangle=\langle f^{-1}\rangle\circ\langle g^{-1}\rangle=\langle f^{-1}\circ g^{-1}\rangle$. □