Proof. By theorem 4.165. □

Proposition 4.228. $(\mathscr{P}\mho)/\sim$ is a boolean lattice.

Proof. By corollary 4.166. □

Proposition 4.229. For a lattice $\mathfrak{F}$ of filters on a set and $a, b \in \mathfrak{F}$ the following expressions are always equal:

1. $a \setminus^* b = \bigsqcap \{z \in \mathfrak{F} \mid a \sqsubseteq b \sqcup z\}$ (quasidifference of a and b);
2. $a \# b = \bigsqcup \{z \in \mathfrak{F} \mid z \sqsubseteq a \wedge z \sqcap b = 0\}$ (second quasidifference of a and b);
3. $\bigsqcup (\operatorname{atoms} a \setminus \operatorname{atoms} b)$.

Proof. Theorem 4.167. □

Conjecture 4.230. $a \setminus^* b = a \# b$ for arbitrary filters a, b on powersets is not provable in ZF (without axiom of choice).

4.4.1 Fréchet Filter

The consideration below is about filters on a set $\mho$, but this can be generalized for filters on complete atomic boolean algebras due complete atomic boolean algebras are isomorphic to algebras of sets on some set $\mho$.

Definition 4.231. $\Omega = \{\mho \setminus X \mid X \text{ is a finite subset of } \mho\}$ is called either *Fréchet filter* or *cofinite filter*.

It is trivial that Fréchet filter is a filter.

Proposition 4.232. $\operatorname{Cor} \Omega = 0^{\mathfrak{P}}$; $\bigcap \Omega = \emptyset$.

Proof. This can be deduced from the formula $\forall \alpha \in \mho \exists X \in \Omega : \alpha \notin X$. □

Theorem 4.233. $\max \{\mathcal{X} \in \mathfrak{F} \mid \operatorname{Cor} \mathcal{X} = 0^{\mathfrak{P}}\} = \max \{\mathcal{X} \in \mathfrak{F} \mid \bigcap \mathcal{X} = \emptyset\} = \Omega$.

Proof. Due the last proposition, it is enough to show that $\operatorname{Cor} \mathcal{X} = 0^{\mathfrak{P}} \Rightarrow \mathcal{X} \sqsubseteq \Omega$ for every filter $\mathcal{X}$.

Let $\operatorname{Cor} \mathcal{X} = 0^{\mathfrak{P}}$ for some filter $\mathcal{X}$. Let $X \in \Omega$. We need to prove that $X \in \mathcal{X}$.

$X = \mho \setminus \{\alpha_0, ..., \alpha_n\}$. $\mho \setminus \{\alpha_i\} \in \mathcal{X}$ because otherwise $\alpha_i \in \uparrow^{-1} \operatorname{Cor} \mathcal{X}$. So $X \in \mathcal{X}$. □

Theorem 4.234. $\Omega = \bigsqcup^{\mathfrak{F}} \{x \mid x \text{ is a non-trivial ultrafilter}\}$.

Proof. It follows from the facts that $\operatorname{Cor} x = 0^{\mathfrak{P}}$ for every non-trivial ultrafilter x, that $\mathfrak{F}$ is an atomistic lattice, and the previous theorem. □

Theorem 4.235. Cor is the lower adjoint of $\Omega \sqcup^{\mathfrak{F}} -$.

Proof. Because both Cor and $\Omega \sqcup^{\mathfrak{F}} -$ are monotone, it is enough (theorem 2.98) to prove (for every filters $\mathcal{X}$ and $\mathcal{Y}$)

$$\mathcal{X} \sqsubseteq \Omega \sqcup^{\mathfrak{F}} \operatorname{Cor} \mathcal{X} \quad \text{and} \quad \operatorname{Cor}(\Omega \sqcup^{\mathfrak{F}} \mathcal{Y}) \sqsubseteq \mathcal{Y}.$$

$\operatorname{Cor}(\Omega \sqcup^{\mathfrak{F}} \mathcal{Y}) = \operatorname{Cor} \Omega \sqcup^{\mathfrak{P}} \operatorname{Cor} \mathcal{Y} = 0^{\mathfrak{P}} \sqcup^{\mathfrak{P}} \operatorname{Cor} \mathcal{Y} = \operatorname{Cor} \mathcal{Y} \sqsubseteq \mathcal{Y}$.

$\Omega \sqcup^{\mathfrak{F}} \operatorname{Cor} \mathcal{X} \sqsupseteq \operatorname{Edg} \mathcal{X} \sqcup^{\mathfrak{F}} \operatorname{Cor} \mathcal{X} = \mathcal{X}$. □

Corollary 4.236. $\operatorname{Cor} \mathcal{X} = \mathcal{X} \setminus^* \Omega$ for every filter on a set.

Proof. By theorem 2.115. □

Corollary 4.237. $\operatorname{Cor} \bigsqcup^{\mathfrak{F}} S = \bigsqcup^{\mathfrak{F}} \langle \operatorname{Cor} \rangle S$ for any set S of filters on a powerset.