Proof. By theorem 4.116. □

Proposition 4.194. Let $\mathfrak{F}$ be the poset of filters on a powerset. $A \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} S = \bigsqcup^{\mathfrak{F}} \langle A \sqcap^{\mathfrak{F}} \rangle S$ for every $A \in \mathfrak{P}$ and every set $S \in \mathscr{P}\mathfrak{F}$.

Proof. By theorem 4.118. □

Proposition 4.195. If S is a generalized filter base of a filter $\mathcal{F}$ on a set $\mathfrak{U}$ then for any $K \in \mathscr{P}\mathfrak{U}$

$$K \in \mathcal{F} \Leftrightarrow \exists \mathcal{L} \in S: K \in \mathcal{L}.$$

Proof. By theorem 4.121. □

Proposition 4.196. If S is a generalized filter base of a filter $\mathcal{F}$ on a set $\mathfrak{U}$ then

$$0^{\mathfrak{F}} \in S \Leftrightarrow \mathcal{F} = 0^{\mathfrak{F}}.$$

Proof. By corollary 4.122. □

Proposition 4.197. Let S be a nonempty set of filters on a set such that $\mathcal{F}_0 \sqcap^{\mathfrak{F}} \ldots \sqcap^{\mathfrak{F}} \mathcal{F}_n \neq 0^{\mathfrak{F}}$ for every $\mathcal{F}_0, \ldots, \mathcal{F}_n \in S$. Then $\bigsqcap^{\mathfrak{F}} S \neq 0^{\mathfrak{F}}$.

Proof. By theorem 4.123. □

Proposition 4.198. Let $S \in \mathscr{P}\mathfrak{U} \setminus \{\emptyset\}$ where $\mathfrak{U}$ is a set and $A_0 \cap \ldots \cap A_n \neq \emptyset$ for every $A_0, \ldots, A_n \in S$. Then $\bigsqcap^{\mathfrak{F}} \langle \uparrow \rangle S \neq 0^{\mathfrak{F}}$.

Proof. By corollary 4.124. □

Proposition 4.199. ∂a is a free star for each filter a on a set.

Proof. By theorem 4.125. □

Proposition 4.200. For a filter $\mathcal{A}$ on a set: $X \in \operatorname{up} \mathcal{A} \Leftrightarrow \overline{X} \notin \partial \mathcal{A}$ for every $X \in \mathfrak{P}$, $\mathcal{A} \in \mathfrak{F}$.

Proof. By theorem 4.126. □

Proposition 4.201. For a filter $\mathcal{A}$ on a set:

1. $\partial \mathcal{A} = \{ \overline{X} \mid X \in \mathfrak{P} \setminus \operatorname{up} \mathcal{A} \}$;
2. $\operatorname{up} \mathcal{A} = \{ \overline{X} \mid X \in \mathfrak{P} \setminus \partial \mathcal{A} \}$

(where complement is taken on the boolean lattice $\mathfrak{P}$).

Proof. By corollary 4.127. □

Proposition 4.202. ∂ is an injection for filters on sets.

Proof. By corollary 4.128. □

Proposition 4.203. For filters on a set: for any set $S \in \mathscr{P}\mathfrak{P}$ there exists a filter $\mathcal{A}$ such that $\partial \mathcal{A} = S$ iff S is a free star.

Proof. By theorem 4.129. □

Proposition 4.204. $\mathcal{A} \sqsubseteq \mathcal{B} \Leftrightarrow \partial \mathcal{A} \subseteq \partial \mathcal{B}$ for every filters $\mathcal{A}$, $\mathcal{B}$ on a set.

Proof. By proposition 4.130. □

Proposition 4.205. ∂ is a straight monotone map for filters on a set.

Proof. By corollary 4.131. □